Biomechanics 1

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Contents

Velocity

- velocity in one dimension: the rate of change of the x coordinate

$$v_x(t) := \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} =: \frac{\mathrm{d}x}{\mathrm{d}t}$$

- SI units of velocity: $[v] = 1\,\frac{\mathrm{m}}{\mathrm{s}} = 3.6\,\frac{\mathrm{km}}{\mathrm{h}}$
- velocity in three dimensions: the rate of change of the position vector **r** – can be constructed from three one-dimensional components

$$\mathbf{v}(t) := \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} =: \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = (v_x(t), v_y(t), v_z(t))$$

Speeds in the human body

Phenomenon	speed
Walking	1.45 m / s
Running (Usain Bolt)	10.44 m / s
Propagation of AP[1] (myelinated fibres)	3–120 m / s
Propagation of AP[1] (non-myelinated fibres)	0.5–2 m / s
Blood flow (aorta)	20–25 cm / s
Blood flow (capillaries)	0.3–0.6 mm / s

[1] action potential

Acceleration

- acceleration: the rate of change of the velocity vector $\mathbf{v}$
- in one dimension:

$$a_x(t) := \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} =: \frac{\mathrm{d}v_x}{\mathrm{d}t}$$

- in three dimensions:

$$\mathbf{a}(t) := \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} =: \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = (a_x(t), a_y(t), a_z(t))$$

- its SI unit: $[a] = 1\,\mathrm{m/s^2}$
- often it is expressed as a multiple of the acceleration due to gravity g ($\approx 9.81\,\mathrm{m/s^2}$)

Acceleration values

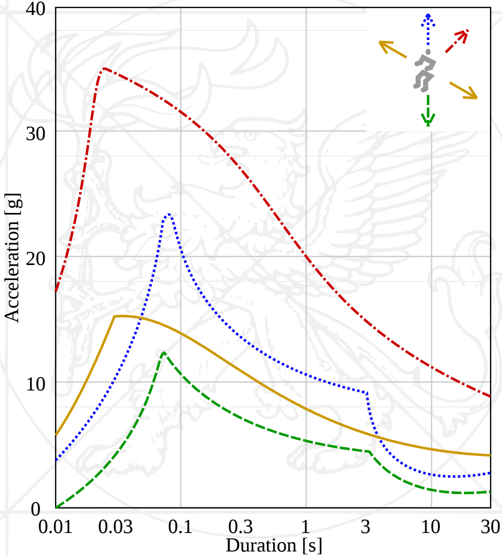
Phenomenon	acceleration
Trabant	$0.12g$
Bugatti	$1.55g$
Formula-1 car, braking	$5.4g$
Roller coaster	$3.5 - 6.3g$
Apollo 16 on reentry	$7.19g$
Death or serious injury likely	$> 25g$
Highest recorded ever survived	$214g$
Handgun bullet	$60\,000g$
Proton in LHC	$1.9 \cdot 10^8 g$

Effects on the human body

Effects on the human body

- speed itself has no effect (relativity principle: systems in uniform motion relative to each other are equivalent)
- mechanism: in an accelerating system, inertial forces will act $\Rightarrow$ organs, blood
- congestion, at other places shortage of blood; deformation of organs
- symptoms:
  - grey-out
  - tunnel vision (loss of peripheral vision)
  - blackout
  - loss of consciousness
  - death

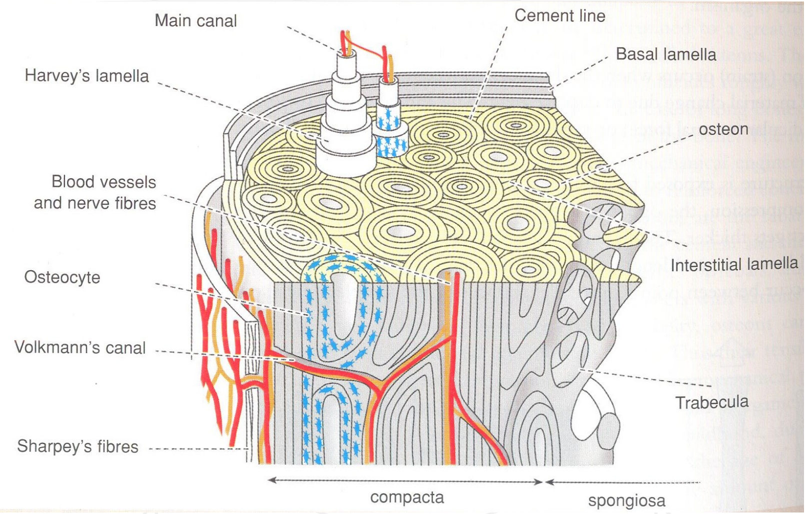
Human acceleration tolerance [3]



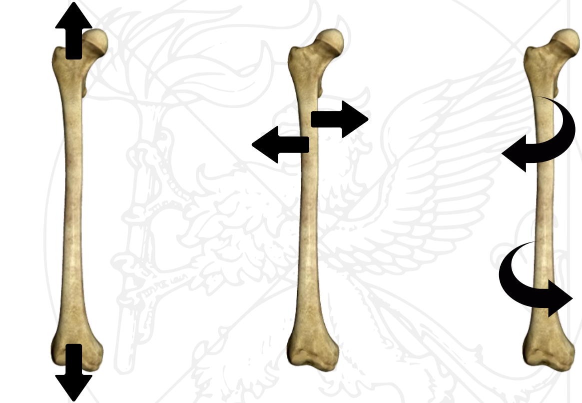
Structure of the bone

- cells (1–5%) + structural component
- composition: $^1/_3$ water + $^1/_3$ organic material (collagen) + $^1/_3$ inorganic material (hydroxyapatite, $Ca_{10}(PO_4)_6(OH)_2$)
- structural unit: osteon – tubular blocks, 5–10 mm in length and 0.2 mm in diameter
- osteon types according to the orientation of collagen fibres:
  - L: longitudinal
  - T: transverse
  - A: alternate left and right spiral
- like steel bars in concrete → elasticity
- can be heavily loaded with tensile and compression forces

Structure of the bone



Deformations



Tension | compression

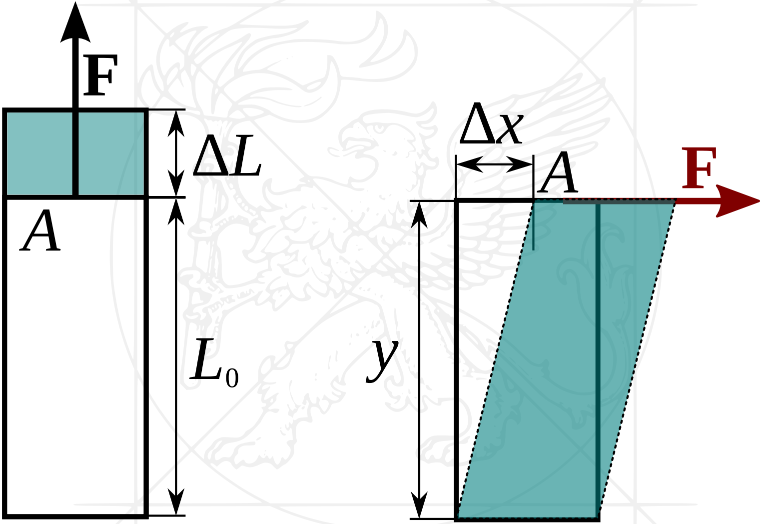
Shearing

Torsion

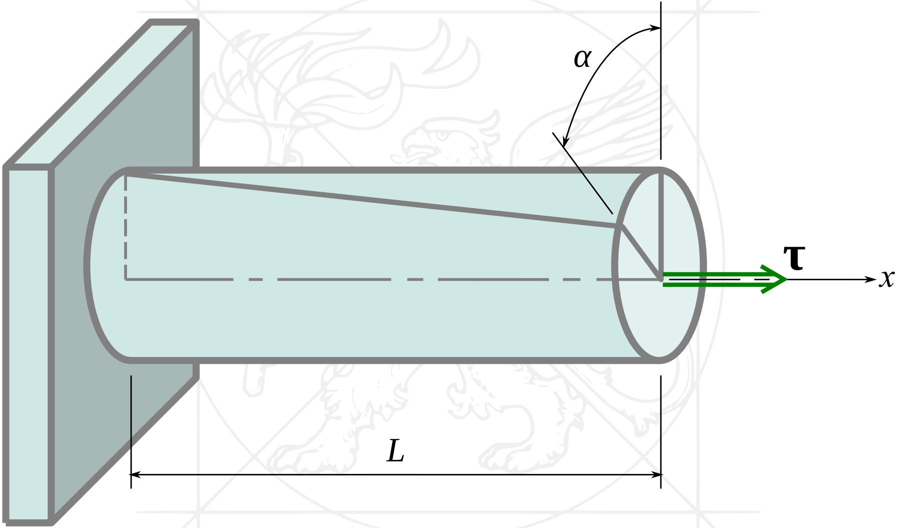
Deformations

- tension | compression: the force causing the deformation is perpendicular to the displaced surface
- shearing: the force causing the deformation is parallel to the displaced surface
- torsion: a torque causes the deformation; the surfaces are not displaced but rotated – the farther they are from the fixed end, the more

Tension and shearing



Torsion



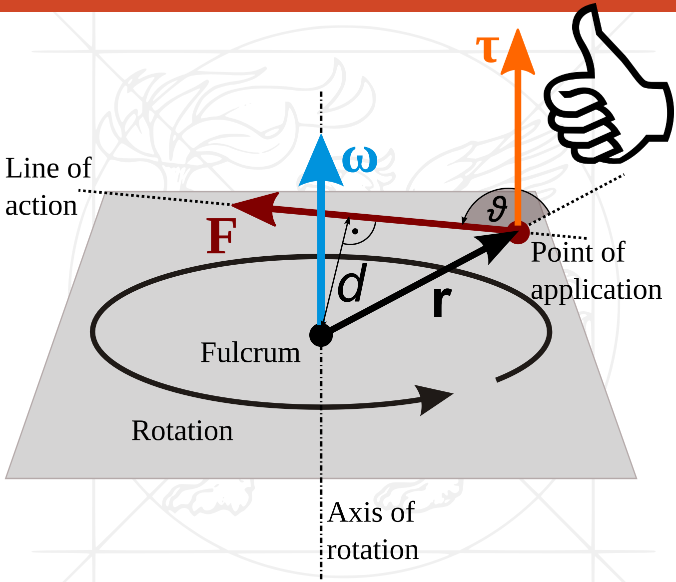
Torque

- torque (τ): the measure of the rotating effect of a force
- lever arm (d): the perpendicular distance of the line of action of a force from the axis of rotation

$$\tau = Fd = Fr\sin\vartheta$$

- SI unit: $[\tau] = 1\,\mathrm{N\,m} \neq 1\,\mathrm{J}$

Torque as a vector



Torque as a vector

- torque vector: the vector product of the position vector from the fulcrum to the point of application of the force and the force itself

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

- if the thumb of the right hand points to the direction of the torque vector, the other fingers curl in the direction of rotation

Elasticity

- strain (ε); for tension or compression $\varepsilon = \Delta L / L_0$
- stress (σ): the ratio of the force causing the deformation to the area of the surface displaced

$$\sigma = \frac{F}{A}, \quad \text{SI unit: } [\sigma] = 1\,\frac{\mathrm{N}}{\mathrm{m}^2} = 1\,\mathrm{Pa}$$

- elastic solids: when the external forces cease to act, they regain their original shape and volume

Hooke's law

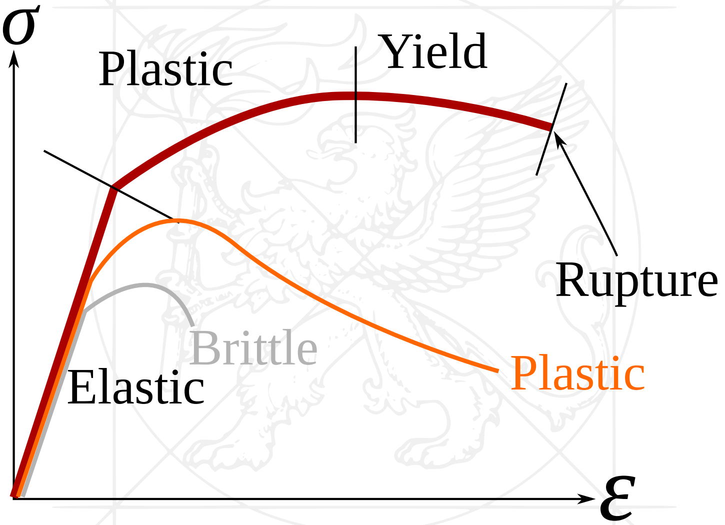
- for relatively low strains, under the so-called limit of proportionality, stress is proportional to strain
- elastic modulus: the constant of proportionality
- Hooke's law (for tension | compression):

$$\sigma = E\varepsilon$$

- E : Young's modulus
- special case: spring law

$$F = \sigma A = EA\frac{\Delta L}{L_0} = k\Delta L$$

Elastic limits



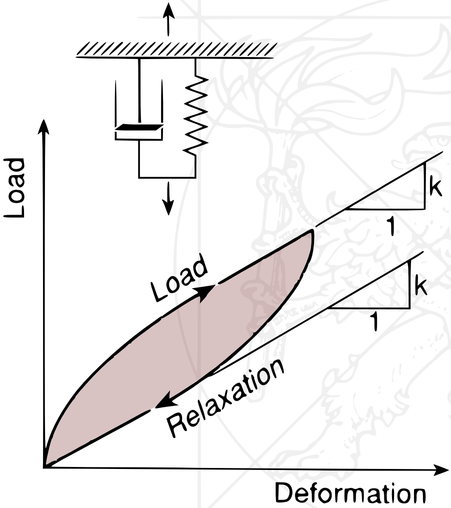
Elastic limits

- elastic limit: the maximum stress for which the elastic solid does not suffer permanent deformation
- limit of proportionality: the maximum stress for which Hooke's law is still valid
- tear strength: the ratio of the force required for fracture to the original cross section

Viscoelasticity

- external forces cause not only elastic deformation but also permanent deformation, depending on how long they act
- after each load-relaxation cycle, the tissues retain some of the energy; after relaxation, they will not return to their original state
- model: Kelvin body – a spring (Hooke body) connected in parallel to a cylinder containing some fluid with a piston (Newton body)
- the Newton body slows down both deformation and relaxation; it consumes energy
- in addition to bones, the ligaments attached to them also have viscoelasticity

Viscoelasticity



Fatigue

- even stresses much less than the tear strength can cause permanent damage if they act repeatedly
- we apply a repetitive force and count how many cycles are needed to cause damage
- empirical law: if $5 \cdot 10^6$ cycles do not break the material, it is deemed permanently resistant to fatigue
- the fibrous architecture of bones slows down the spreading of fatigue-induced fracture; the laminar structure of osteons stops cracks
- march fracture: fatigue-induced fracture on the metatarsal bones of marching soldiers

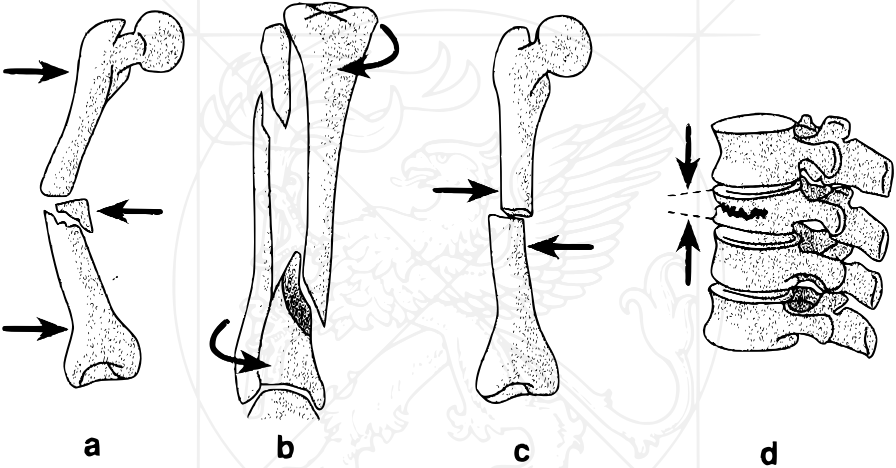
Physical investigation of bone tissue

- hardness test: pressing a very hard tip of defined geometry perpendicularly into the sample; the surface of the dent is then assessed
- fracture test: we determine the fracture limit with a clockwork pendulum; decreasing the applied force on subsequent samples
- load-deformation analysis: we record the stress-strain curve and determine the elastic limit, tear strength, *&c*
- using modern methods, elastic properties can be determined also on a microscopic scale, on the level of osteons

Elastic properties of bone tissue

- due to osteons, bone structure shows similarity to fibreglass and carbon fibre materials
- microfractures → wound healing mechanisms: bone structure is adapted to the load
- the compressive yield strength of cortical bone is 140 MPa (that of steel is 200 MPa); that of spongiosa is 20 times less
- its tensile strength is nearly as much; its resistance to shear and torsion is much less
- properties of osteons
    - L: resistant mostly to tension
    - T: resistant mostly to compression
    - A: average resistance against every type of load

Fractures



Fractures: a) bending with wedge fracture; b) torsion with spiral fracture; c) shear with transverse fracture; d) compression with impacted fracture [1, p. 369.]

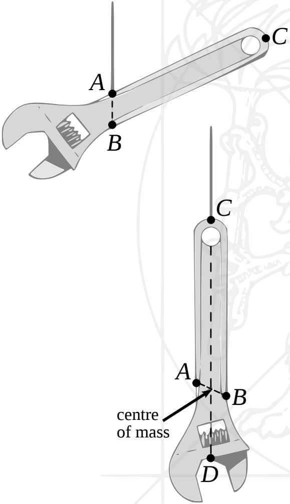
Centre of mass

- the position vector of the centre of mass of a system consisting of N particles:

$$\mathbf{R} = \frac{1}{M}\sum_{k=0}^{N-1} m_k \mathbf{r}_k$$

- the centre of mass of a system behaves as if the whole mass of the system were concentrated there as a particle
- the centre of mass can be considered as the point of application of the force of gravity

Determining the centre of mass

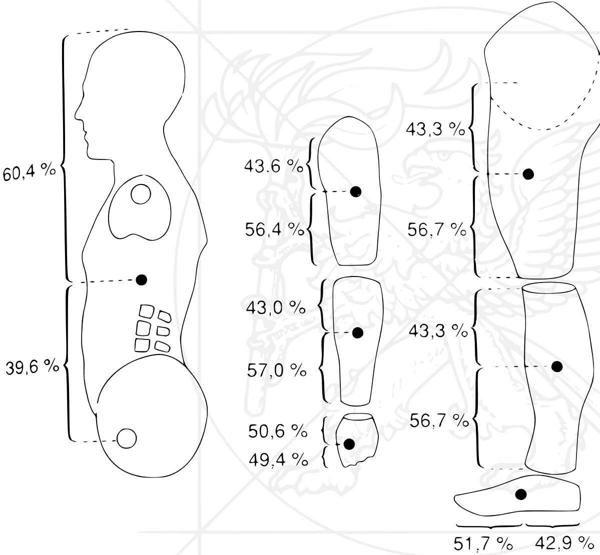


- we hang the object from two different points
- line of gravity: a vertical line passing through the point of suspension
- the centre of mass is located at the intersection of the two lines of gravity

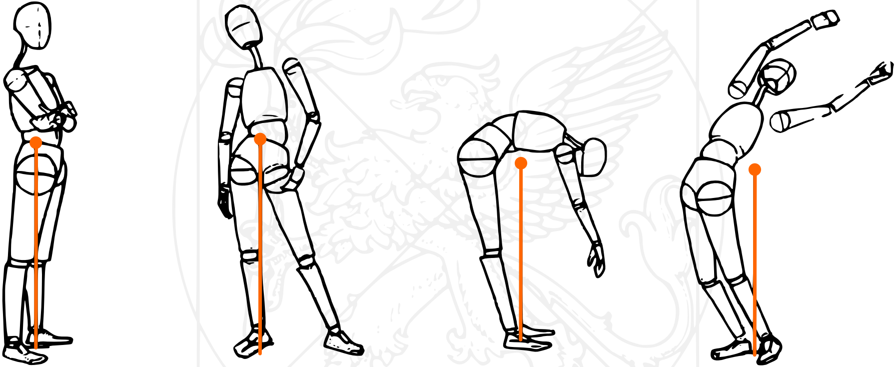
Determining the centre of mass

- we cannot apply the previous method to a human body
- Dempster method: we determine the centre of mass for the body parts individually; the centre of mass of the whole body is the centre of mass of the particle system consisting of the centres of mass of the individual body parts
- capable of taking individual peculiarities into consideration:
    - types of obesity
    - lack of limbs

The Dempster method



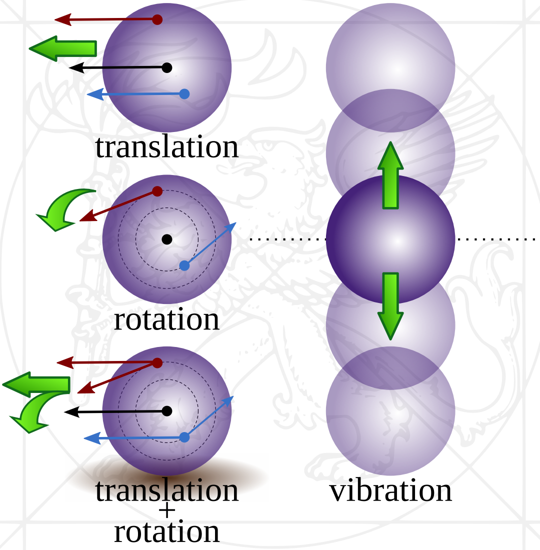
Centre of mass



Statics of the human body

- different postures can only be achieved through the active exertion of muscles
- when one stands on two legs, there exists a posture in which only a minimal muscular force is needed: energy-sparing body position
- every other posture requires more muscle exertion
- double S curvature of the spine: $\rightarrow$ greater supporting elasticity
- a rod with N bends has $(N^2 + 1)$ times greater supporting elasticity than a straight rod

Types of motion



Conditions for equilibrium

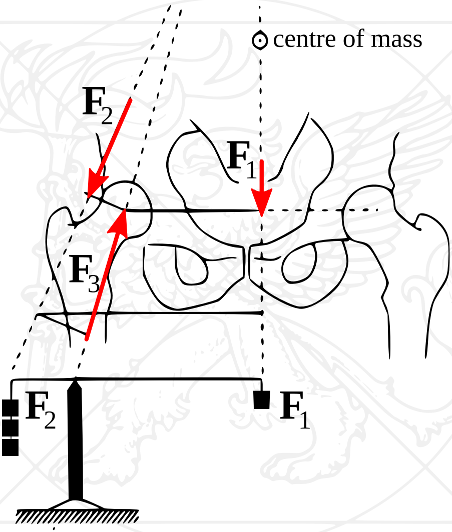
- equilibrium: the state of motion of the body (including translation and rotation) does not change (ie, does not accelerate)
- conditions for equilibrium:
  1. the vector sum of external forces is zero

     $$\sum \mathbf{F} = 0$$

  2. the vector sum of external torques about any axis is zero

     $$\sum \boldsymbol{\tau} = 0$$

Statics of standing on one leg



Statics of standing on one leg

- the weight $\mathbf{F}_1$, whose point of application is in the centre of mass, has a torque that would rotate the torso about the femoral head of the supporting leg
- to counter this, muscles around the hip tighten $\rightarrow$ $\mathbf{F}_2$
- condition for equilibrium for the torques about an axis passing through the femoral head (for simplicity, we approximate the forces as vertical):

$$F_1 d_1 = F_2 d_2$$

- since the lever arm d_1 of the force $\mathbf{F}_1$ is much greater, the magnitude of $\mathbf{F}_2$ must be much greater than that of the weight (about thrice as much)

Statics of standing on one leg

- the condition for equilibrium for the forces acting on the torso balanced on the femoral head must also be met
- for this, a support force $\mathbf{F}_3$ is needed at the femoral head
- balance of the forces:

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$$

- approximating the forces as vertical, this means the following for the magnitude of $\mathbf{F}_3$:

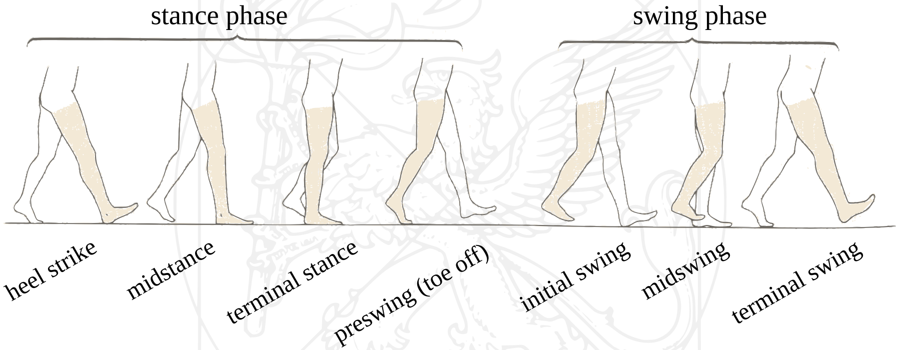
$$F_3 = F_1 + F_2$$

Walking and the centre of mass

Walking and the centre of mass

- if the forces propelling the body forwards do not pass exactly through the centre of mass, torques will act that would rotate the body left or right
- this is critical for quadrupeds (cf galloping horse v monitor lizard), less critical for bipeds
- here the limbs are not located behind and ahead of the centre of mass, but below, close to the line of gravity $\rightarrow$ the lever arm is small
- there still exist lateral torques; to counter this, we keep relocating the centre of mass

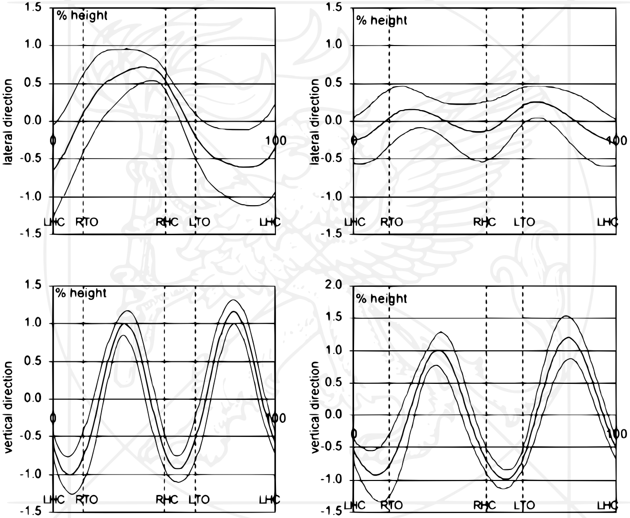
Phases of walking



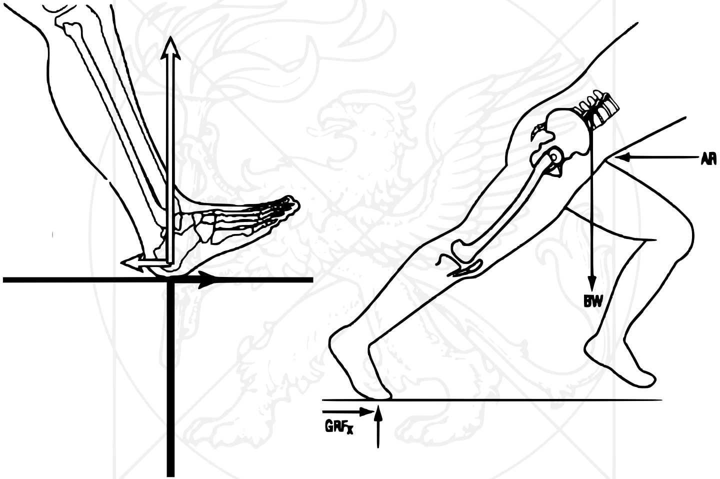
Trajectory of the centre of mass

- energetically, the most efficient way for the centre of mass would be moving in a straight line
- this is not the case in walking: the centre of mass follows a sinusoidal trajectory both vertically and laterally
- this needs extra energy; the greater the displacement, the more
- badly fixed or weighted lower-limb prostheses: the patient can only walk at the expense of significant extra work

Trajectory of the centre of mass



The role of friction in walking



Kinetic friction

- force of friction: a force that acts between surfaces in motion or in impending motion relative to each other
- if the surfaces actually move relative to each other: kinetic friction

$$F_{\mathrm{kf}} = \mu F_{\mathrm{N}}$$

where F_{N} stands for the normal force pressing the surfaces together and μ is the coefficient of kinetic friction

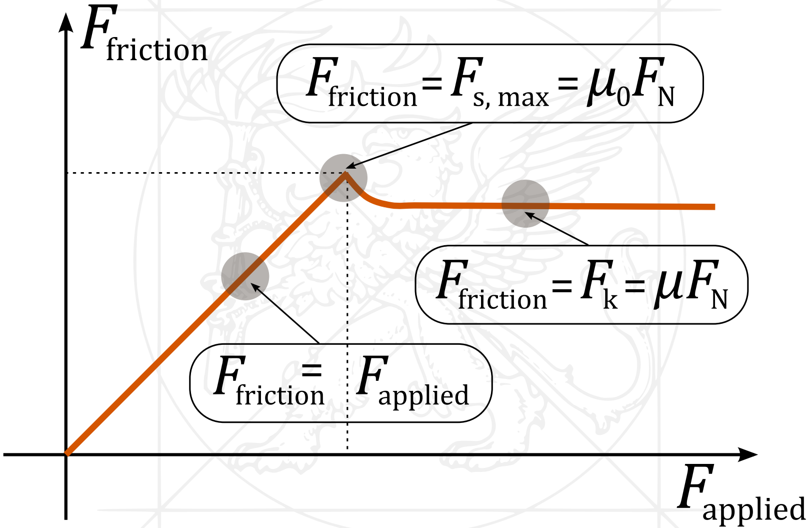
Static friction

- if there is no actual motion, only impending motion (motion would occur if not for friction): static friction
- up to a limit, it is always of the magnitude to stop the surfaces from moving relative to each other
- but it cannot exceed a maximum value

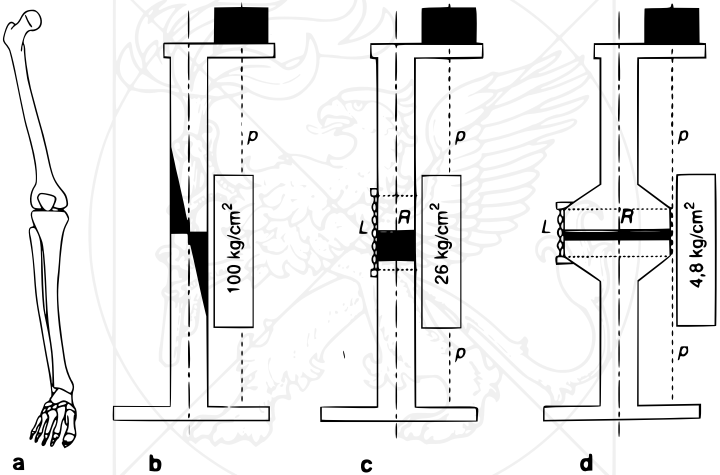
$$F_{\text{sf, max}} = \mu_0 F_{\text{N}}$$

where $\mu_0 > \mu$ is the coefficient of static friction

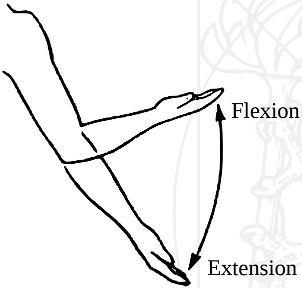
Forces of friction



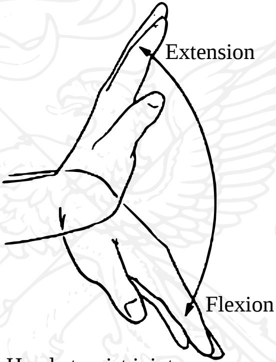
Static load on joints



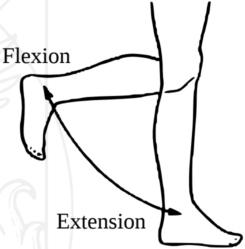
Flexion and extension [4]



Forearm at elbow joint

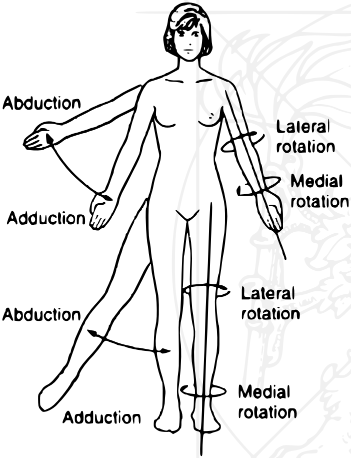


Hand at wrist joint



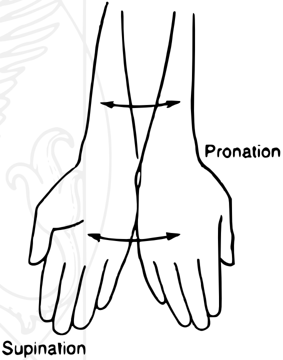
Leg at knee joint

Rotation of joints [4]



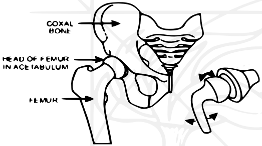
Abduction and adduction of right limbs and rotation of left limbs at glenohumeral and hip joints, respectively

Circumduction (circular movement) of lower limb at hip joint

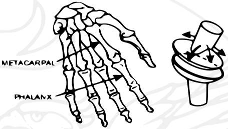


Pronation and supination of forearm at radioulnar joints

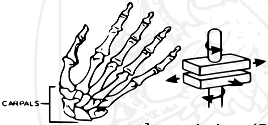
Joint types



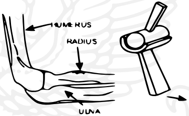
ball-and-socket joint (3D)



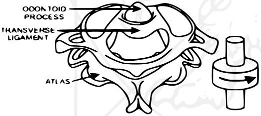
condyloid (ellipsoidal) joint (2D)



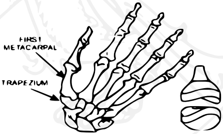
plane joint (2D)



hinge joint (1D)



pivot joint (1D)

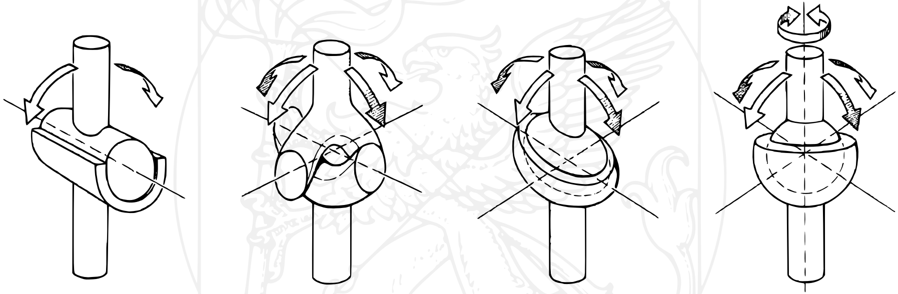


saddle joint (2D)

Degrees of freedom

- degree of freedom: an independent type of motion through which a body can possess energy
- examples:
  - train: 1 degree of freedom
  - car in a parking lot: 2 degrees of freedom
  - aeroplane: 3 degrees of freedom
- there are also degrees of freedom associated with rotation
- as many rotational degrees of freedom as there are independent axes about which the body can freely rotate

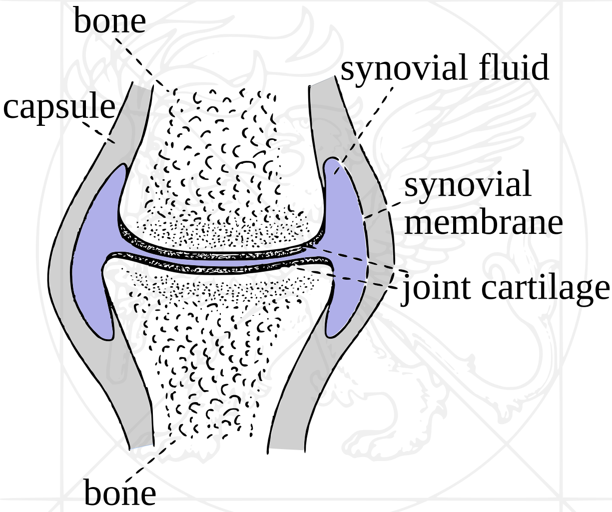
Degrees of freedom for joint types



Tribology

- technical discipline dealing with the damage at the contact surfaces of bodies in relative motion
- mechanisms for wear and surface damage:
  - abrasion
  - adhesion
  - fatigue
  - chemical (eg, corrosive)
- decreasing wear: lubrication
- lubrication in joints: synovial fluid $\rightarrow$ thin liquid film

Schematic structure of a joint



Prostheses

- biomaterials:
  - metals: stainless steel, titanium, alloys of cobalt, chrome, nickel, vanadium and aluminium
  - plastics: polyethylene of ultra-high molecular mass
  - bone cement: polymethyl methacrylate
- anchoring to bone tissue: mechanically (tension or bolts) or cementing with polymethyl methacrylate
- surface treatment: eg evaporating bone material and depositing it upon the surface $\rightarrow$ faster bone formation around the implant

Sources and recommended reading I

[1] Sándor Damjanovich, Judit Fidy, and János Szöllősy, editors. *Medical biophysics*. Medicina, Budapest, 3rd edition, 2009.

[2] Attila Fonyó et al. *Principles of Medical Physiology*. Medicina, Budapest, 2002.

[3] https://commons.wikimedia.org/wiki/File:Human_linear_acceleration_tolerance.svg.

[4] Irving P Herman. *Physics of the Human Body*. Springer-Verlag, Berlin–Heidelberg, 2007.

Sources and recommended reading II

[5] T Jurcevic Lulic and O Muftic. Trajectory of the human body mass centre during walking at different speed. https://www.designsociety.org/download-publication/29655/trajectory_of_the_human_body_mass_centre_during_walking_at_different_speed.