Optics in medicine

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Contents

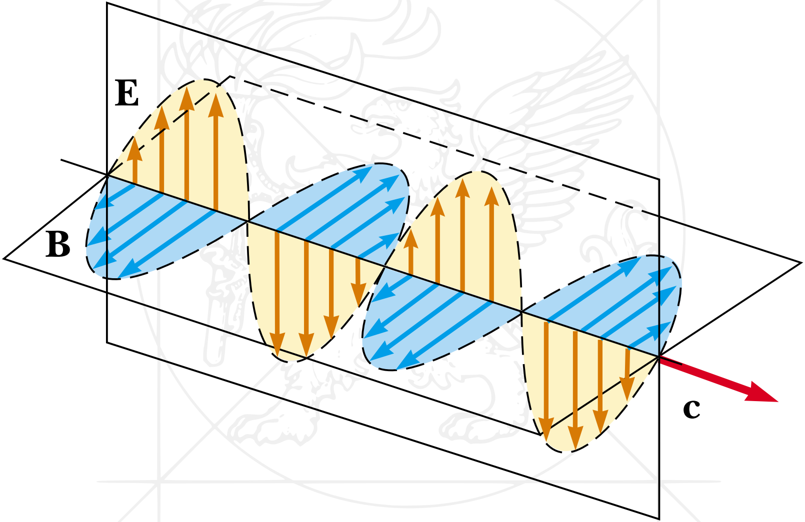
The nature of light

- What is light? A stream of particles as NEWTON thought, or a wave?
- since the beginning of the 19th century, light is conceived of as a transverse wave
- MAXWELL, 1873: there are electromagnetic waves, and light is one form of them
- quantum mechanics in the 20th century: light has particle aspects as well (photons — light quanta, light ‘particles’)
- visible light: between 390 and 750 nm in wavelength

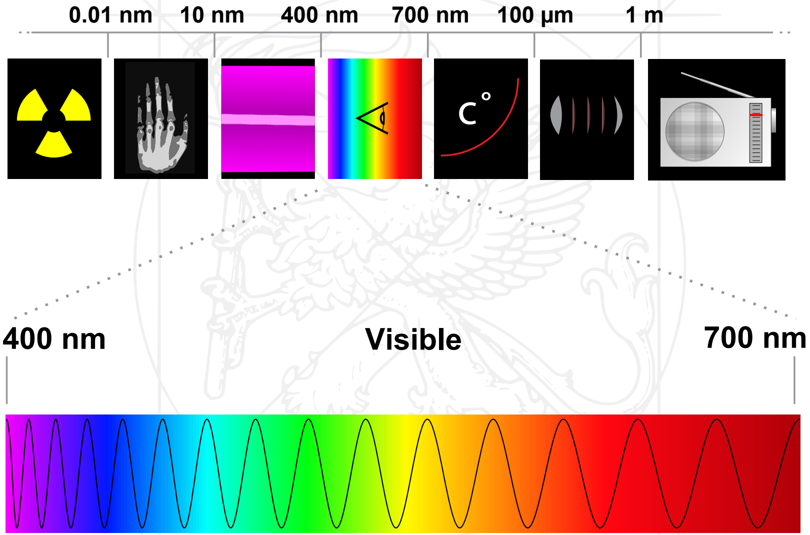
Electromagnetic waves

- the changing electric field produces a changing magnetic field ⇒ the changing magnetic field produces a changing electric field ⇒ the changing electric field produces a changing magnetic field ⇒ the changing magnetic field produces a changing electric field ⇒ the changing magnetic field . . .
- **electromagnetic waves** (light, X-rays, *&c*)
- transverse waves
- do not require a medium to propagate

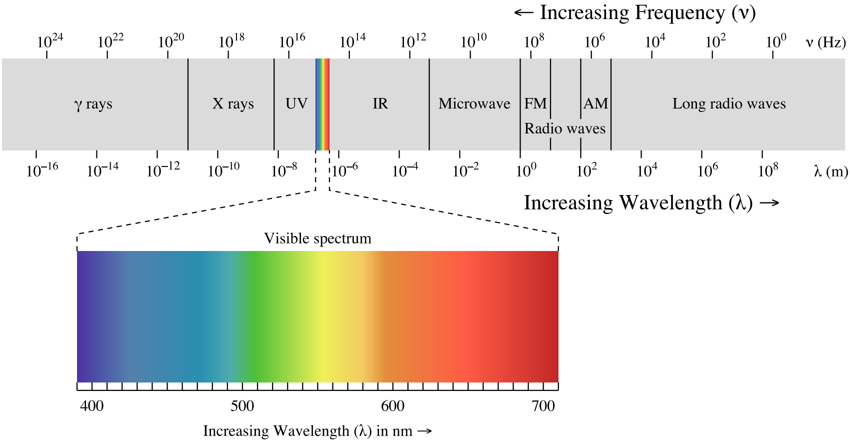
Electromagnetic waves



The electromagnetic spectrum



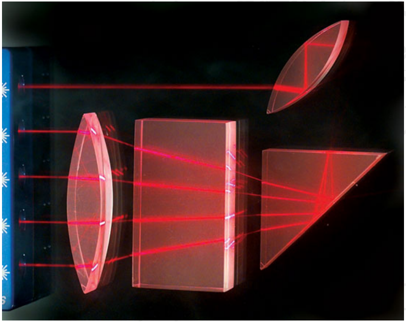
The electromagnetic spectrum



Approaches

Geometrical optics

diffraction and interference effects negligible; straight propagation until boundary



Wave optics

diffraction and interference play a role

Light as a stream of photons

- photoelectric effect: the energy of the electrons ejected from a metal illuminated by light depends on the frequency and not on the intensity of light
- Einstein's explanation: light consists of energy quanta (energy packets) called **photons**
- photon energy:

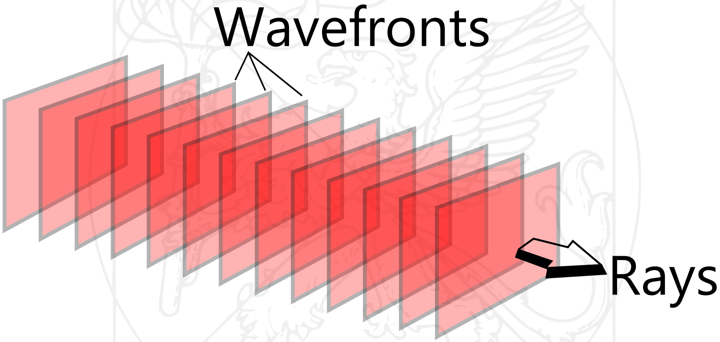
$$E = h\nu,$$

where ν denotes the frequency and $h \approx 6.63 \cdot 10^{-34}$ Js is **Planck's constant**

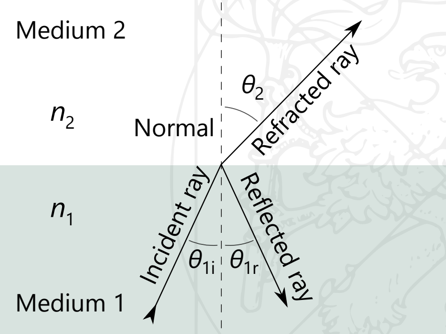
The ray model of light

- ***ray model of light:*** the assumption that light travels in a fixed direction in a straight line as it passes through a uniform medium and changes its direction when it meets the surface of a different medium or if the optical properties of the medium are non-uniform in either space or time
- ***geometrical optics:*** optics based on the ray model of light
- ***wave front:*** a surface that connects points of equal phase (eg, crests) on all waves
- ***rays:*** straight lines perpendicular to the wave fronts

Rays and wavefronts

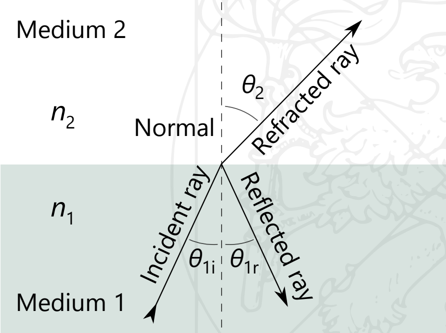


The law of reflexion



- whenever light reaches a boundary between two media, part of it will be reflected
- ***normal:*** the line perpendicular to the surface at the point where the incident light strikes the surface

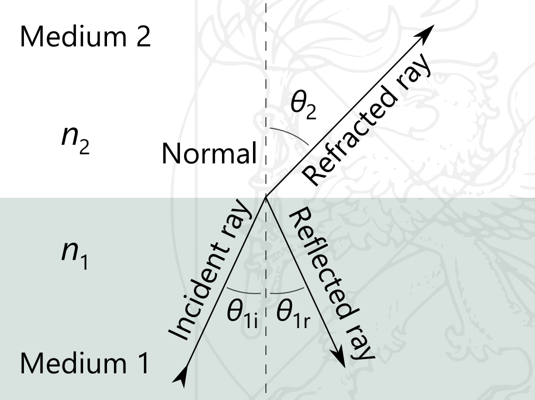
The law of reflexion



The law of reflexion

1. the incident ray, the reflected ray and the normal are **in the same plane**
2. the angle of reflexion is the same as the angle of incidence: $\theta_{1,\mathrm{r}} = \theta_{1,\mathrm{i}}$

Refraction



whenever light reaches a boundary between two media, part of it will enter a new medium, but will change its direction — this is called ***refraction***

The law of refraction or *Snell's law*

1. the incident ray, the refracted ray and the normal are **in the same plane**
2. the ratio of the sine of the angle of refraction to the sine of the angle of incidence equals the ratio of the speed of propagation in the new medium to the speed of propagation in the old medium

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

Refractive index

- ***refractive index*** of a medium: the ratio of the speed of propagation of the wave in a reference medium (eg, vacuum in the case of light), c, to the speed of propagation of the wave in the given medium, v

$$n := \frac{c}{v}$$

- relative refractive index of medium 1 with respect to medium 2:

$$n_{21} := \frac{n_2}{n_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2}$$

Refractive index

- media with higher optical density (that is, in which light propagates slower) have greater refractive index

Substance	air	water	glass	diamond	quartz	ethyl alcohol
Refractive index	1	1.333	1.5–1.6	2.419	1.458	1.361

Snell’s law using refractive indices

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} = n_{21}$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

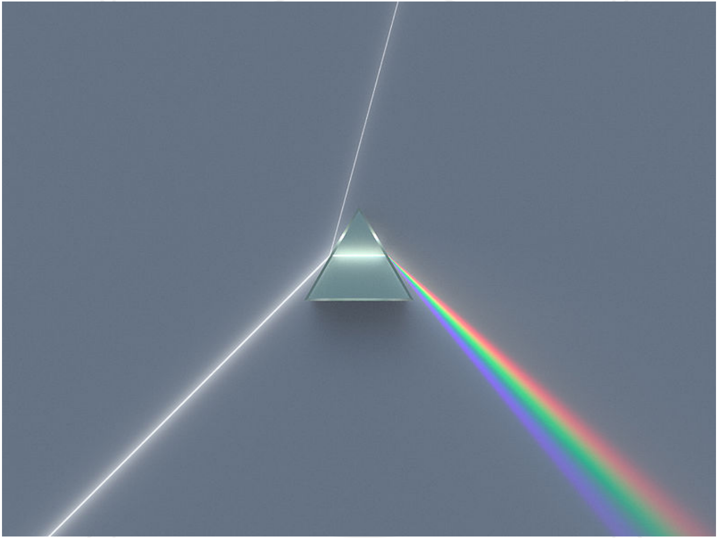
Dispersion

- ***dispersion:*** the refractive index n of a medium is a function of the wavelength ($n = n(\lambda)$), so it will be different for waves with different wavelengths
- as a result of dispersion, waves with different wavelength will be refracted at different angles and will travel different paths
- white light is a mixture of many colours; it can be broken down into these colours by a dispersive element

Dispersion

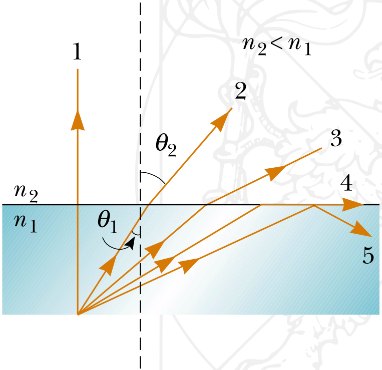
- ***dispersive element:*** an optical device which breaks down white light into its constituent colours, like a **prism** or a **grating**
- example: rainbow — raindrops will refract different colours of light at different angles, so white light is broken down into different colours
- ***monochromatic rays:*** rays that be described by a single wavelength (that is, colour — from Greek ‘single colour’)
- monochromatic rays can not be broken down into further components by a dispersive element; they follow a single path

Dispersion on a prism

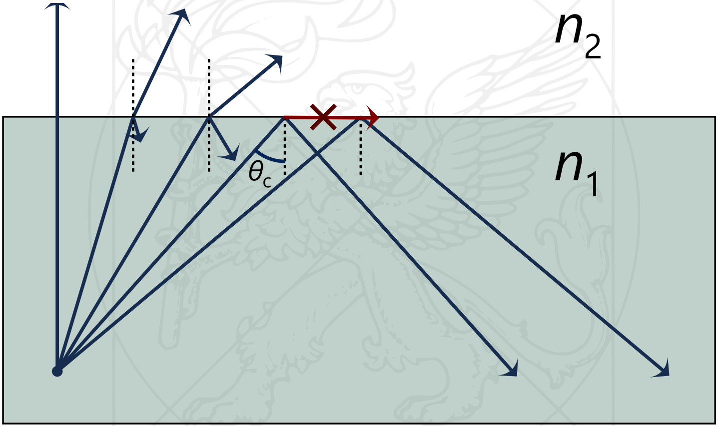


A famous album cover. Whose?

Total internal reflexion



Total internal reflexion



Total internal reflexion

Total internal reflexion

- let us assume that light travels from a medium of refractive index n_1 into an optically less dense medium with refractive index $n_2 < n_1$ (eg, glass $\rightarrow$ air)
- in this case, the angle of refraction will be greater than the angle of incidence
- ***critical angle*** θ_c: an angle of incidence at which the angle of refraction is 90°, so the refracted ray would move parallel to the boundary between the media

Total internal reflexion

$$n_1 \sin\theta_\mathrm{c} = n_2 \sin 90^\circ = n_2$$

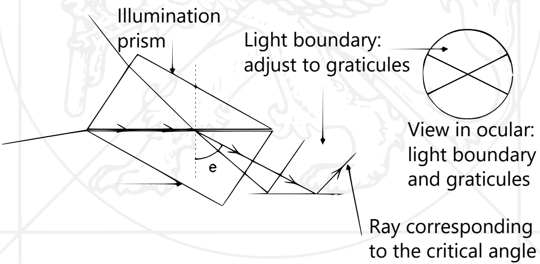
$$\boxed{\sin\theta_\mathrm{c} = \frac{n_2}{n_1}}$$

- What happens when the angle of incidence is greater than θ_c? There will be no refracted ray; the incident ray will be fully reflected.
- ***total internal reflexion:*** when light is directed from a medium towards a medium with a lower refractive index, rays above the critical angle will not be refracted but will be fully reflected

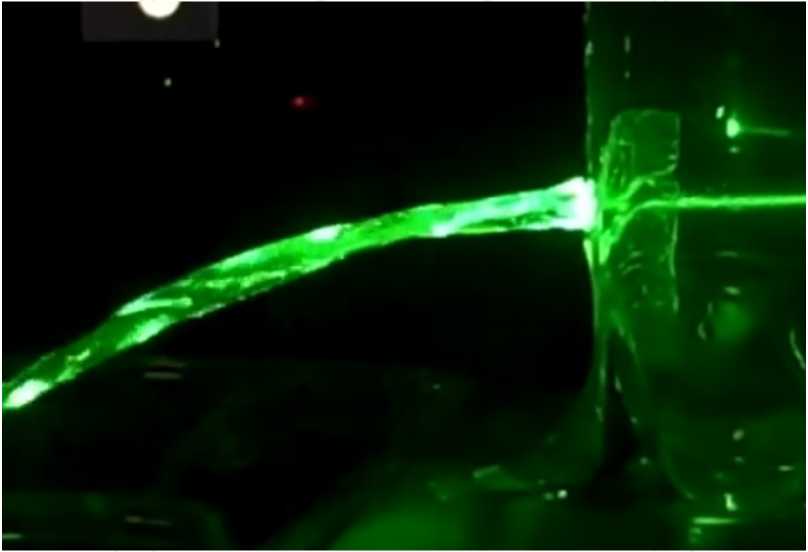
Total internal reflexion: an example

Application: Abbe refractometer

- two-prism arrangement to determine the refractive index of solutions
- the solution is placed between the prisms
- uses the principle of total internal reflexion
- medical applications: to determine the concentration of plasma proteins in blood



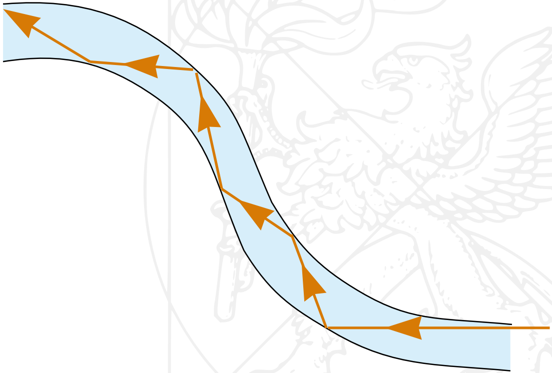
The principle of optical fibres

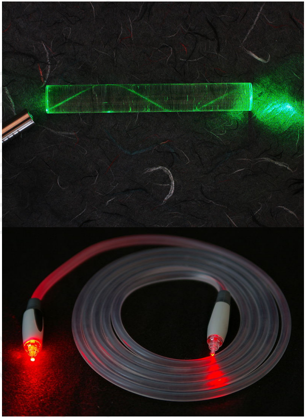


Optical fibres

- **optical fibres** use total internal reflexion: because they are thin, the angle of incidence is guaranteed to be above the critical angle, so total internal reflexion will occur and no light can leave the fibre through the walls of the fibre, even if the fibre is curved or even coiled

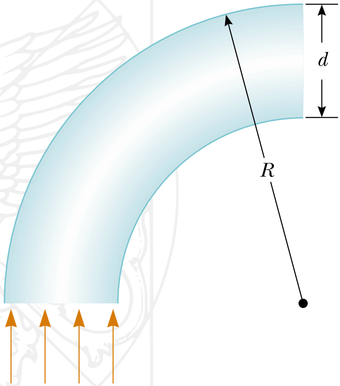
Optical fibres





Optical fibres

An optical fibre has index of refraction n and diameter d. It is surrounded by air. Light is sent into the fibre along its axis, as shown in the figure. Find the smallest outside radius R permitted for a bend in the fibre if no light is to escape.

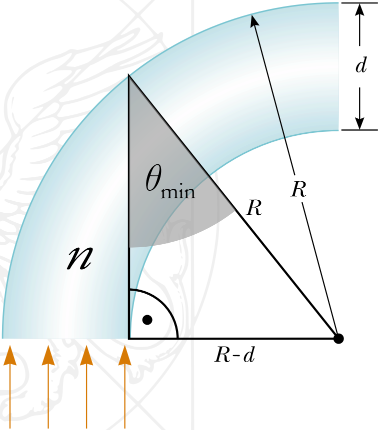


Optical fibres

- the innermost ray will have the smallest angle of incidence, thus the greatest chance of escaping; for this ray (assuming $n_{\mathrm{air}} = 1$)

$$\sin\theta_{\min} = \frac{R-d}{R} \geq \sin\theta_{\mathrm{c}}$$

$$\sin\theta_{\min} \geq \sin\theta_{\mathrm{c}} = \frac{1}{n}$$

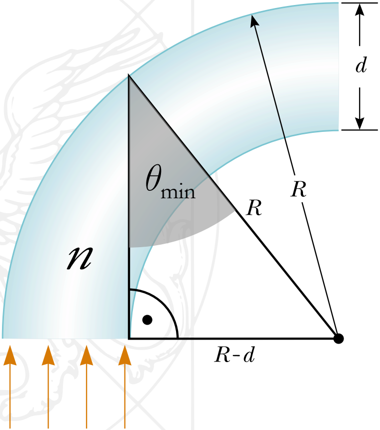


Optical fibres

- rearranging this, we get

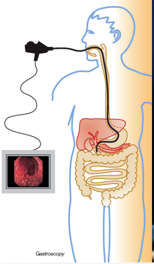
$$R \geq \frac{dn}{n-1} = \frac{d}{1-1/n},$$

☛ the thinner the fibre and the greater its index of refraction, the more we can bend it (that is, the smaller the bend radius permitted) without any loss of light



Optical fibres in medicine

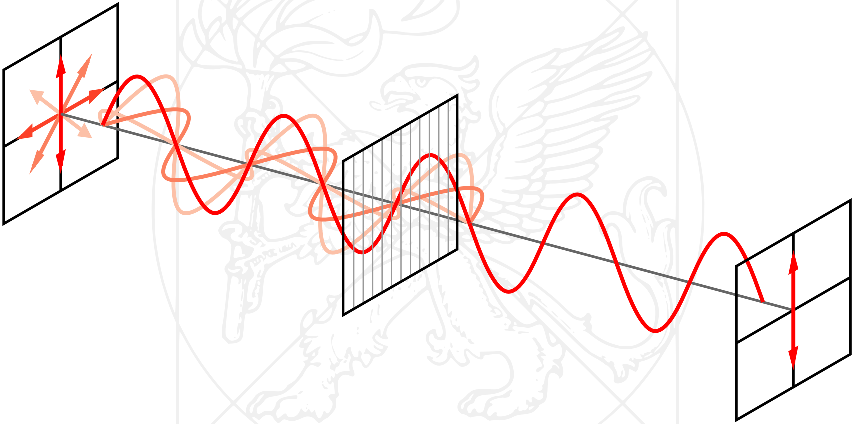
- steering light into and out of an internal cavity
- image formation: endoscopy
- intervention: guiding laser light to the operation area; dentistry, laser surgery



Polarisation

- transverse waves: oscillations are perpendicular to the direction of propagation
- there are many planes perpendicular to the direction of propagation in which oscillations can take place
- ***polarisation*** is a property of waves describing the orientation of oscillations
- for example, linear polarisation: oscillations take place in a given plane
- ***polarisers:*** devices which transmit waves only with a given polarisation; when used to analyse light polarisation, they are called ***analysers***

Polariser and polarisation



Polarisation of reflected light

- light coming directly from a source has usually no dominant polarisation
- light reflected from a surface is more polarised
- ☛ reflected light can be filtered out using an appropriately orientated analyser: **polarisation filter**

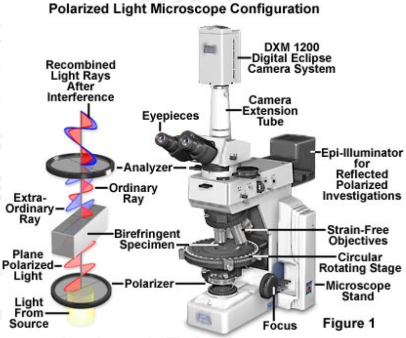
Polarisation filters

Optical birefringence

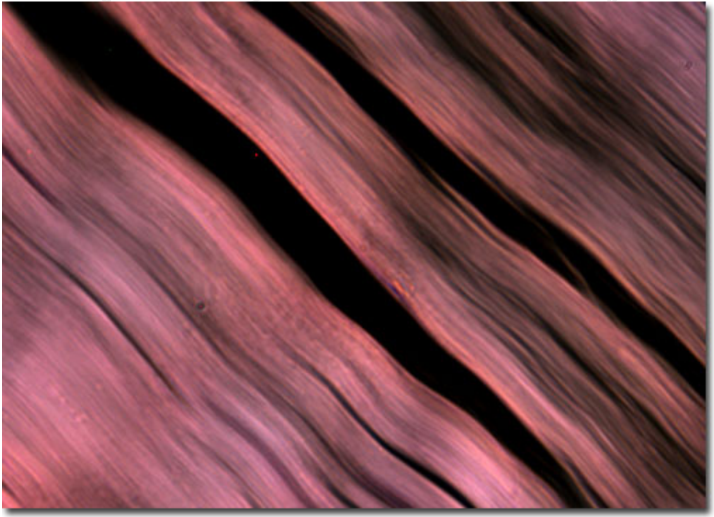
- two orthogonally polarised beams (ie, beams whose polarisation planes are perpendicular to each other) are refracted with different refractive index in certain crystals
- ☛ two orthogonally polarised images

Polarisation microscopy

- microscopic techniques that utilise polarisation
- used especially for birefringent samples: eg, A-bands of sarcomeres



Human striated muscle



Basic concepts

- **object:** the real-world object we view through an optical device (mirror or lens) — denoted by **O**
- **image:** what the mirror or lens produces — denoted by **I**
- images are found by **extending diverging rays to a point at which they intersect**
- ***real image:*** light rays actually pass through a point and diverge from there
- ***virtual image:*** light rays appear to diverge from a point they never actually pass through

Basic concepts

- ***object distance*** (p): the distance of the object from the mirror or lens
- ***image distance*** (q): the distance of the image from the mirror or lens
- **object size** (h): the size of the object in a dimension perpendicular to the axis of the mirror or lens
- **image size** (h'): the size of the image in the same direction in which the object size is measured
- ***magnification:*** the ratio of image size to object size:

$$M := \frac{h'}{h}$$

Image formation of a plane mirror

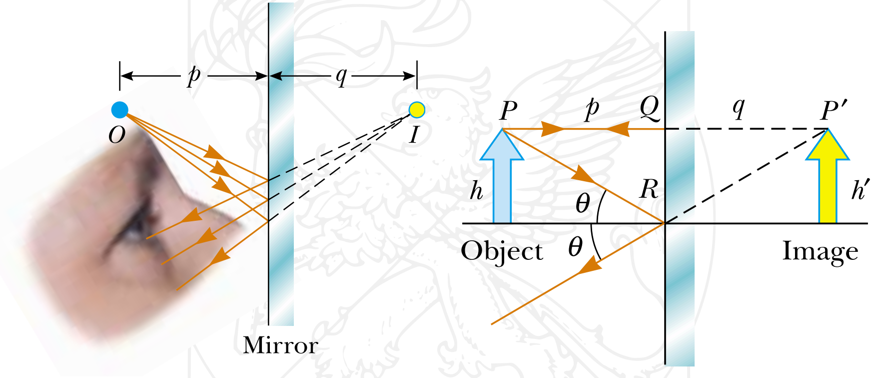


Image formation of a plane mirror

Plane mirrors

- for all mirrors, **image formation is based on reflexion**
- the image appears behind the mirror
- **the image is upright**
- **the image is virtual**

Plane mirrors

- because of the law of reflexion, triangles $\triangle \mathrm{PQR}$ and $\triangle \mathrm{P'QR}$ are congruent $\Rightarrow$ their corresponding sides are equal:

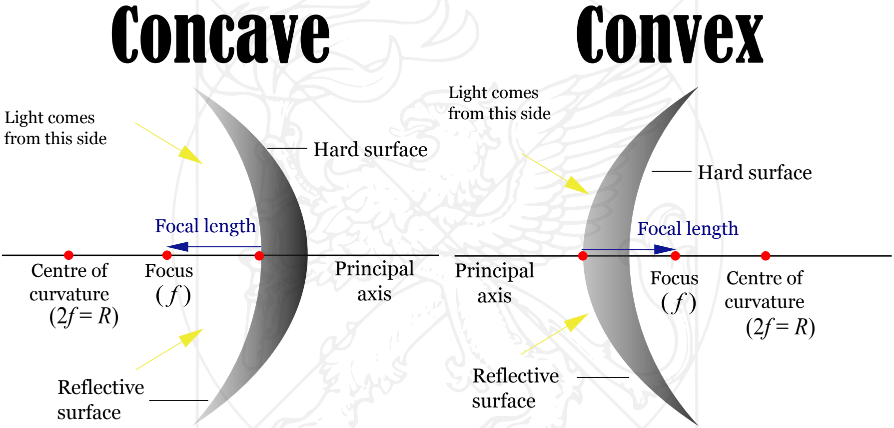
$$\mathrm{P'Q} = \mathrm{PQ} \;\Rightarrow\; |q| = |p|$$

$$h' = h$$

- image size is equal to object size; the image distance is equal to object distance
- the magnification is 1

$$M = \frac{h'}{h} = -\frac{q}{p} = 1$$

Spherical mirrors



Spherical mirrors

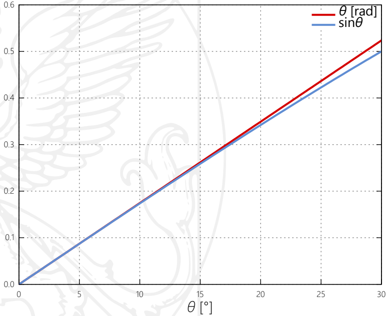
- **principal axis:** the line passing through the centre of the sphere from which the mirror was cut and attaching to the mirror in the exact centre of the mirror
- **centre of curvature** (C): the centre of the sphere from which the mirror was cut
- **radius of curvature** R: the radius of the sphere from which the mirror was cut

Spherical mirrors

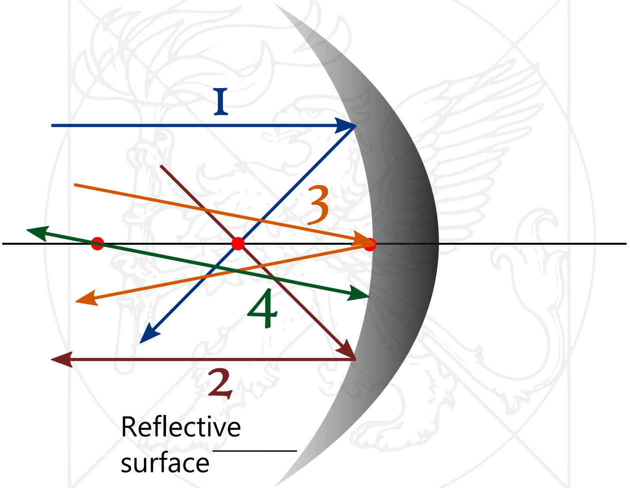
- **vertex** (V) the point where the principal axis meets the mirror
- **focal point** (F): the point exactly in midway between the vertex and the centre of curvature
- **focal length** f: the distance between the vertex and the focal point

Paraxial rays

- the relationships to follow are approximations
- only valid for rays travelling close to the principal axis and forming a small angle ($\theta \leq 5°$) with it – ***paraxial rays***
- for paraxial rays, the sine of the angle can be approximated by the angle itself in radians



Principal rays for concave mirrors



Principal rays for concave mirrors

1. any incident ray travelling **parallel** to the principal axis will **pass through the focal point** upon reflexion
2. any incident ray **passing through the focal point** will travel **parallel** to the principal axis upon reflexion
3. the incident ray that reaches the mirror **at the vertex** will be reflected **at an angle equal to the angle of incidence**
4. any incident ray passing through the centre of curvature will pass through the centre of curvature also upon reflexion

Sign conventions for mirrors

Quantity	Positive when	Negative when
object location p	object in front of mirror (real object)	object in back of mirror (virtual object)
image location q	image in front of mirror (real image)	image in back of mirror (virtual image)
image height h'	upright image	inverted image
focal length f	concave mirror	convex mirror
magnification M	upright image	inverted image

Image formation of a concave mirror

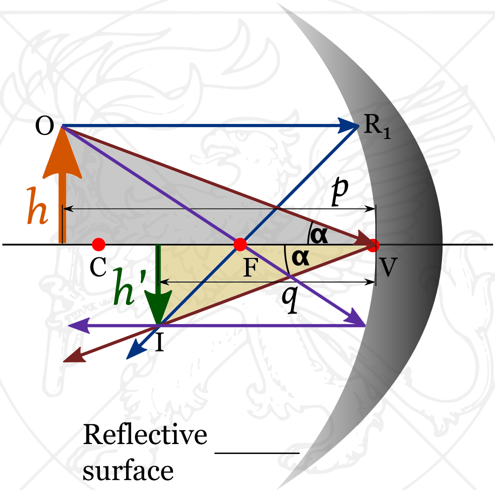


Image formation of a concave mirror

Image formation of a concave mirror

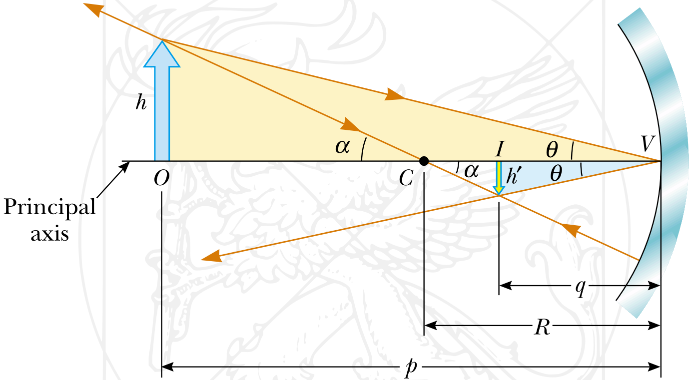


Image formation of a concave mirror

The mirror equation

- comparing the golden and the blue triangle we can see that

$$\tan\theta = \frac{h}{p} = -\frac{h'}{q}$$

- the negative sign of h' is because the image is inverted
- from the definition of the magnification, it follows that

$$M = \frac{h'}{h} = -\frac{q}{p} \tag{1}$$

The mirror equation

- comparing the right-angled triangles that have C as a common vertex we can see that

$$\tan\alpha = \frac{h}{p-R} = -\frac{h'}{R-q}$$

- rearranging that, we get

$$\frac{h'}{h} = -\frac{R-q}{p-R}$$

The mirror equation

- comparing this to Equation (1), we get

$$\frac{q}{p} = \frac{R-q}{p-R}$$

- rearranging this, we get

$$\frac{1}{q} + \frac{1}{p} = \frac{2}{R}$$

- the focal length is equal to half the radius of curvature, so

$$\frac{2}{R} = \frac{1}{f}$$

The mirror equation

- so finally we got the mirror equation, which links the focal length to the object and image distance

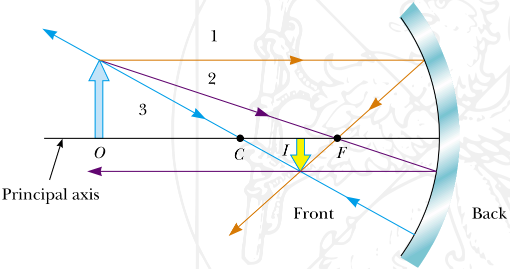
$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

- it is true for both convex and concave mirrors with the appropriate sign conventions

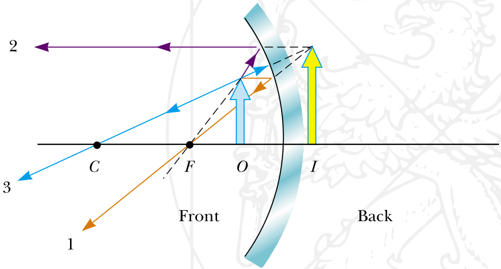
Image properties

Object position	Type	Orientation	Magnification
$p > R$	real	inverted	reduced: $-1 < M < 0$
$p = R$	real	inverted	same size: $M = -1$
$f < p < R$	real	inverted	magnified: $M < -1$
$p < f$	virtual	upright	magnified: $M > 1$
$p = f$	no image	—	—

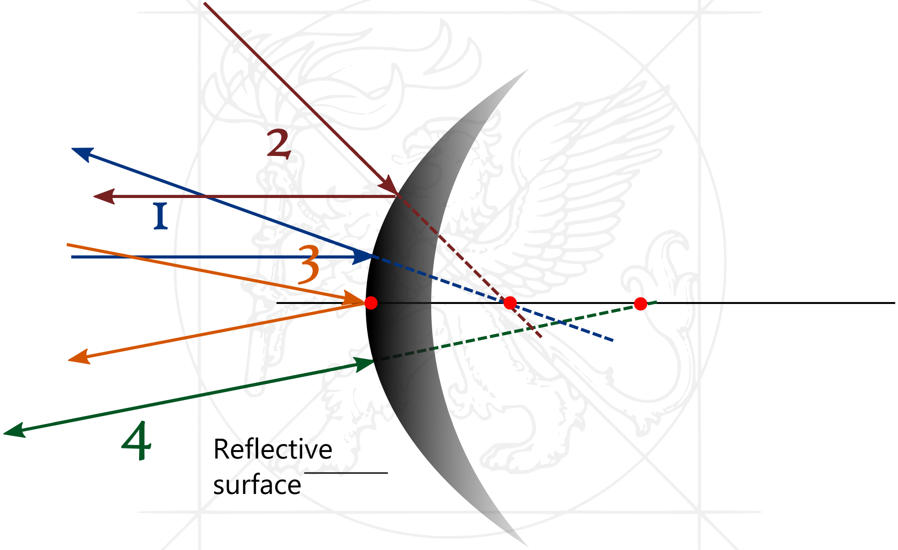
Object outside the centre



Object inside the focal point



Principal rays for convex mirrors



Principal rays for convex mirrors

1. any incident ray travelling **parallel** to the principal axis will be reflected so that its extension **will pass through the focal point**
2. any incident ray whose extension **passes through the focal point** will be reflected **parallel** to the principal axis
3. the incident ray that reaches the mirror **at the vertex** will be reflected **at an angle equal to the angle of incidence**
4. any incident ray whose extension passes through the centre of curvature will pass through the centre of curvature also upon reflexion

Image properties for convex mirrors

the image is always **virtual, upright and reduced** $(|M| < 1)$

Image formation of a convex mirror

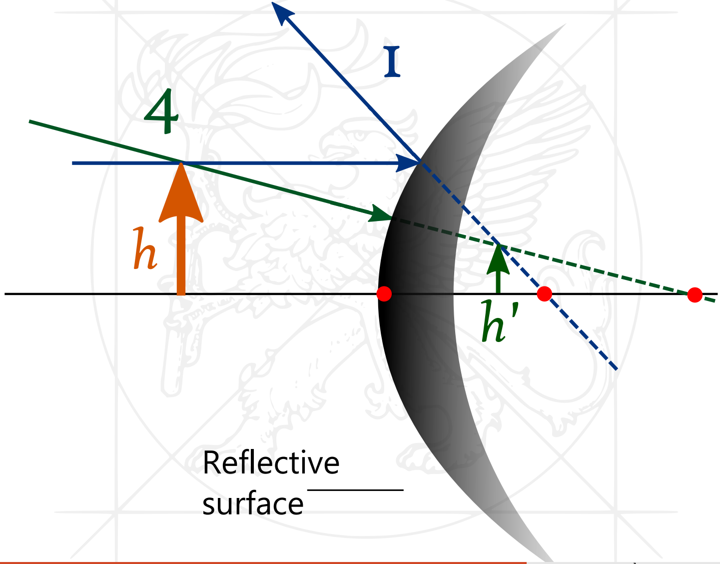
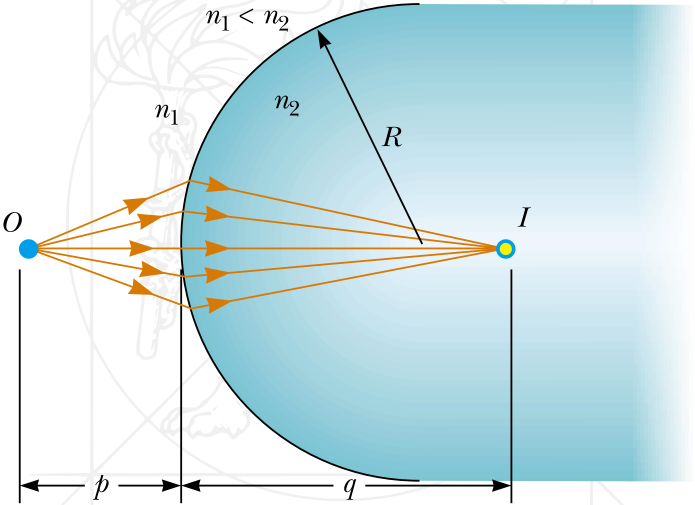
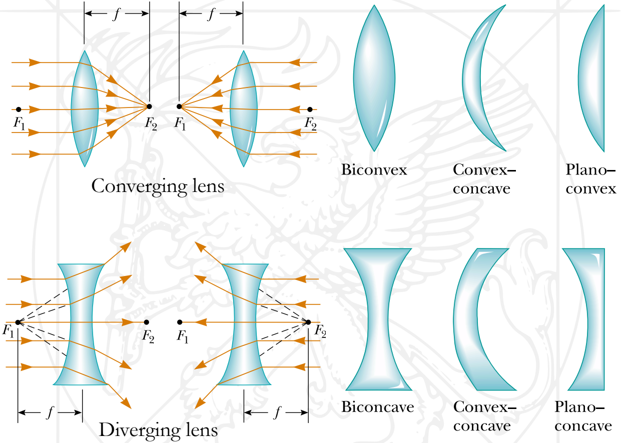


Image formation by refraction



Lens types



Since light can pass through the lens in both directions, lenses have focal points on both sides.

Laws of thin lenses

- ***thin lens:*** its thickness is negligible as compared to the radii of curvature
- in a similar way to the mirror equation, we can find a ***lens equation:***

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

- the magnification of a lens can be expressed the same way as that of a mirror:

$$M = \frac{h'}{h} = -\frac{q}{p}$$

- these laws are valid both for converging and diverging lenses

Optical power

- the reciprocal of the focal length of a lens (or curved mirror) is called the ***optical power:***

$$D = \frac{1}{f}$$

- optical power is measured in **dioptre**s (US ‘diopter’); unit symbol: **dpt**

$$1\,\mathrm{dpt} = 1\,\mathrm{m}^{-1}$$

- one **dioptre** is the optical power of a lens with a focal length of one metre

Lens systems

- for systems consisting of thin lenses with focal lengths f_1 and f_2, the focal length of the system will be given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

- in systems of thin lenses, the optical powers are added together:

$$D = D_1 + D_2$$

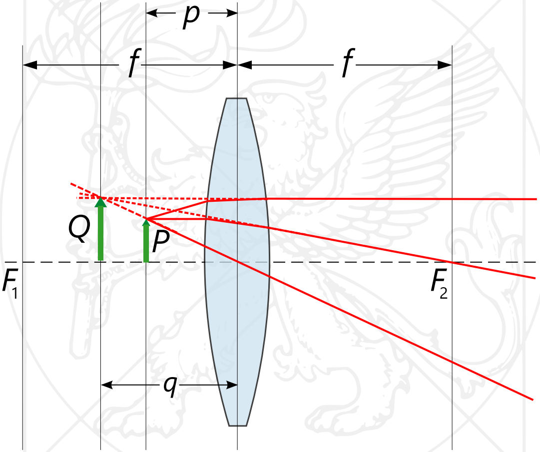
Sign conventions for lenses

Quantity	Positive when	Negative when
object location p	object in front of lens (real object)	object in back of lens (virtual object)
image location q	image in back of lens (real image)	image in front of lens (virtual image)
image height h'	upright image	inverted image
radius of curvature	convex surface	concave surface
focal length f	converging lens	diverging lens
magnification M	upright image	inverted image

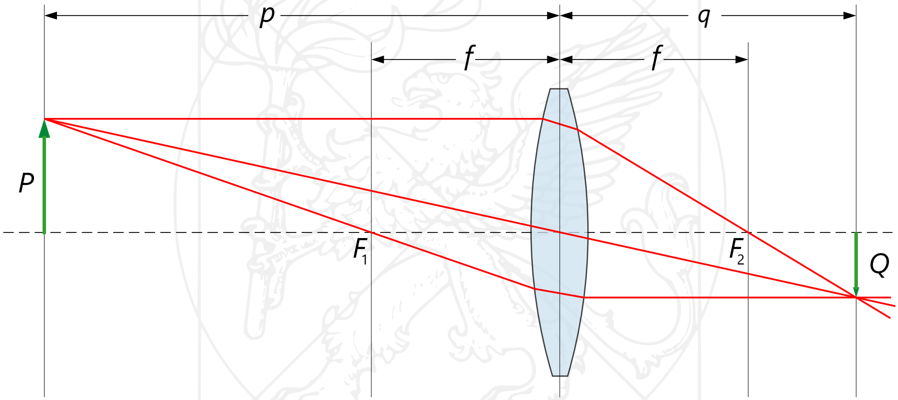
Principal rays for converging lenses

1. any incident ray **parallel** to the principal axis will be refracted so that it will pass **through the focal point on the other side**
2. any incident ray passing through the centre of the lens will continue in the same line
3. any incident ray passing through a focal point will be refracted so that it will travel on parallel to the principal axis

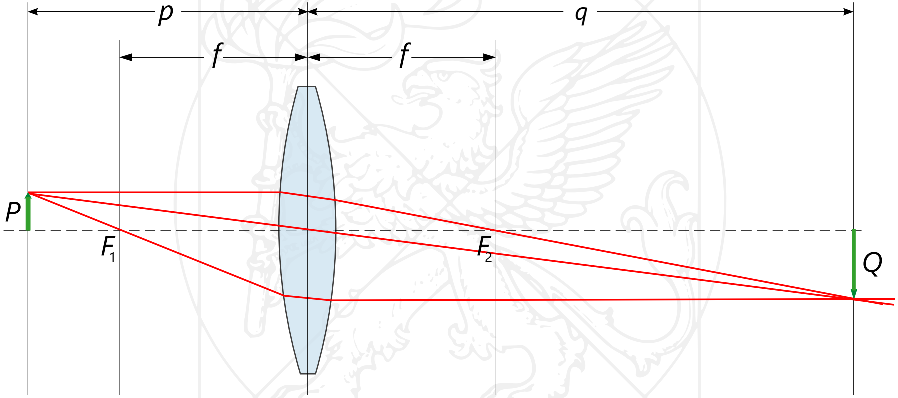
Principal rays for converging lenses



Object outside $2f$



Object between $2f$ & f



Object within f

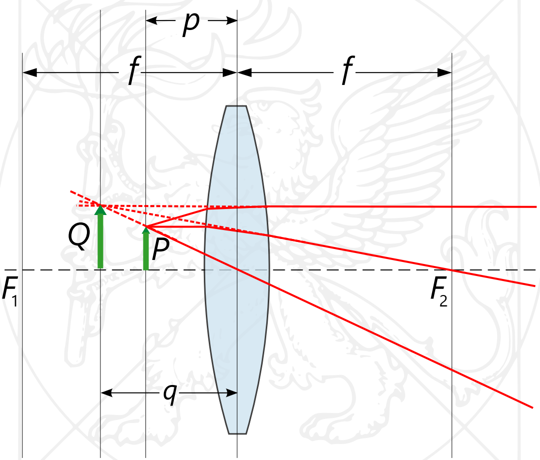


Image properties

Object position	Type	Orientation	Magnification
$p > 2f$	real	inverted	reduced: $-1 < M < 0$
$p = 2f$	real	inverted	same size: $M = -1$
$f < p < 2f$	real	inverted	magnified: $M < -1$
$p < f$	virtual	upright	magnified: $M > 1$
$p = f$	no image	—	—

Principal rays for diverging lenses

1. any incident ray **parallel** to the principal axis will be refracted as if it had passed **through the focal point on the same side**
2. any incident ray passing through the centre of the lens will continue in the same line
3. any incident ray whose extension passes through a focal point on the other side will be refracted so that it will travel on parallel to the principal axis

Principal rays for diverging lenses

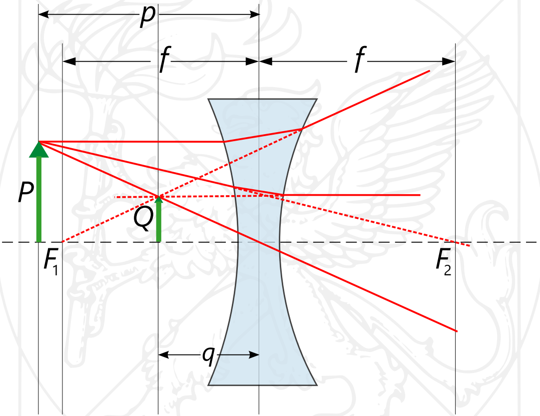


Image properties: diverging lenses

For a diverging lens, the image is always **virtual, upright and reduced.**

Lensmakers’ equation

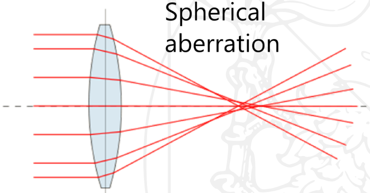
- if the media on the two sides of the lens are identical:

$$D = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

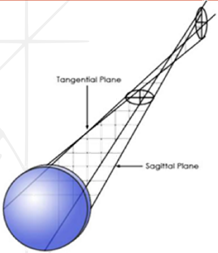
- where n is the refractive index of the lens material relative to the medium and R_1 and R_2 are the radii of curvature of the lens surfaces
- if we have a medium of absolute refractive index n_1 on one side and another of n_2 on the other side, and the absolute refractive index of the lens is n_0:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_0 - n_1}{R_1} + \frac{n_0 - n_2}{R_2}$$

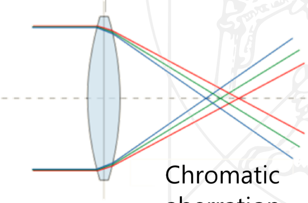
Lens aberrations



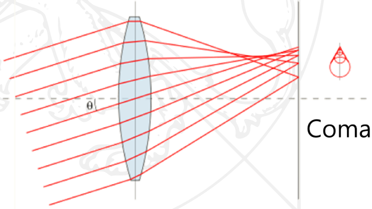
Spherical aberration



Astigmatism



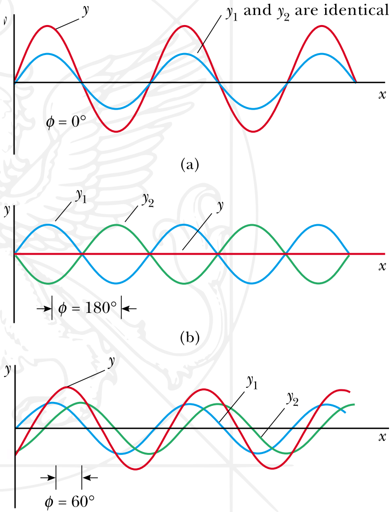
Chromatic aberration



Coma

Superposition

- when two waves meet, their wave functions are added
- constructive interference: crest meets crest
- destructive interference: crest meets trough

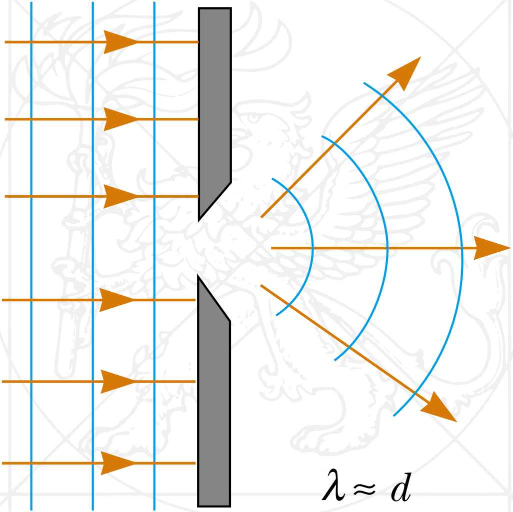


Interference

- **interference:** the waves being superposed create a stable pattern of amplification and cancellation depending on the phase difference between the waves
- this requires that the waves should be **coherent:** the phase relationship between the waves should be constant
- natural light sources are not coherent, but lasers are
- to create holograms, coherent illumination is needed
- grainy appearance of laser dots on a surface: lasers are so coherent that they can show diffraction on not fully regular porous surfaces

Thin-film interference

Diffraction



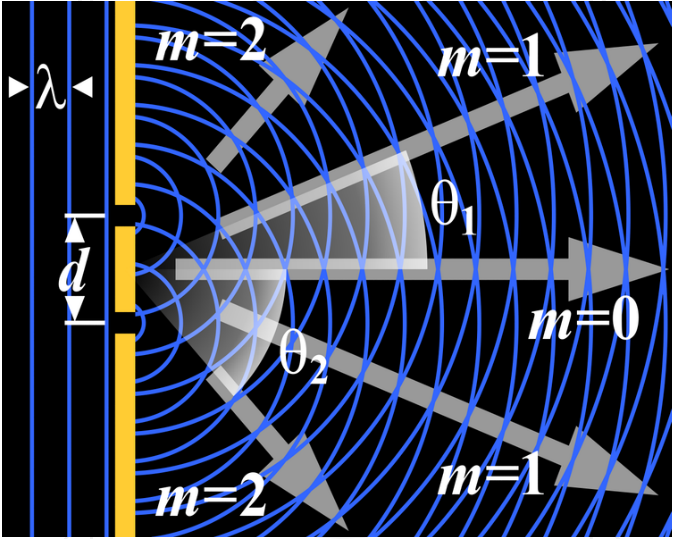
Diffraction

- when a wave encounters an obstacle whose dimensions are in the same order of magnitude as the wavelength, the wave will ‘bend’ — it will travel in other directions in addition to the original one
- this phenomenon is called ***diffraction***

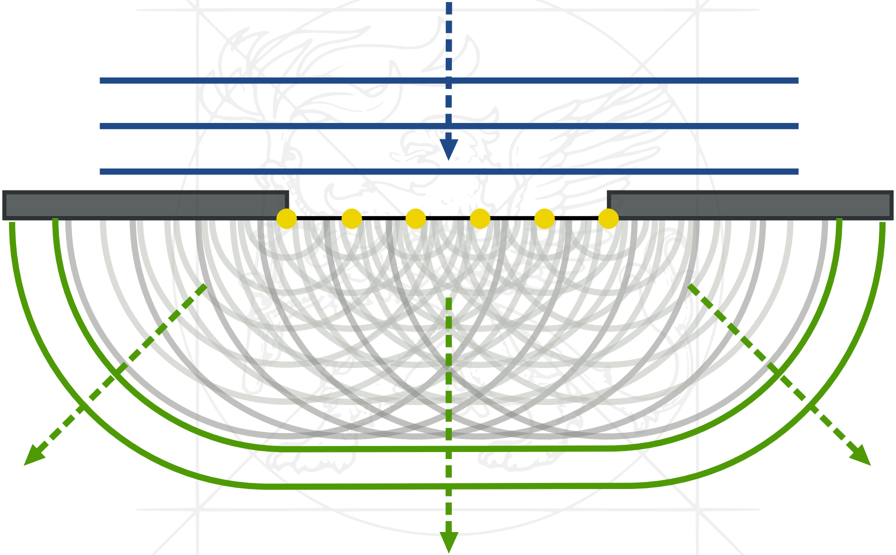
Huygens–Fresnel principle

- ***Huygens–Fresnel principle:*** each point of an advancing wave front is the centre of a new disturbance and the source of a new train of waves; additionally, the advancing wave as a whole may be regarded as the sum of all the secondary waves arising from points in the medium that the wave has already passed
- the Huygens–Fresnel principle can explain reflexion, refraction and diffraction

Diffraction



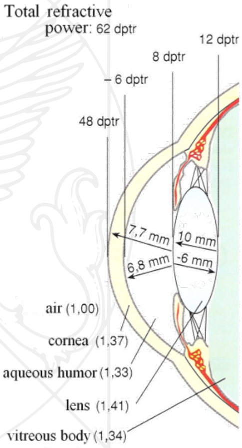
Huygens–Fresnel principle



Refraction in the eye

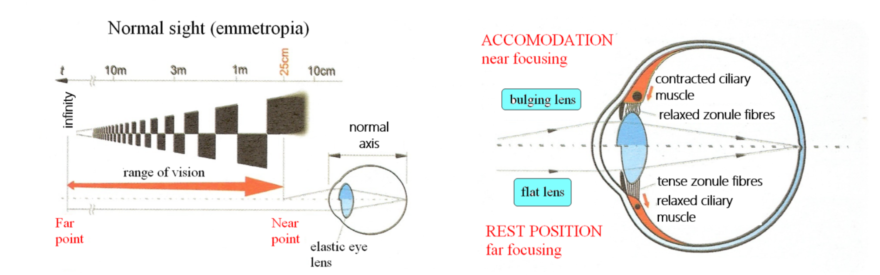
$$D = \frac{1}{f} = \frac{n_2 - n_1}{R}$$

- the optical powers of the individual surfaces add up
- greatest refractive power: air – cornea
- crystalline lens: variable optical power



Accommodation

varying the refractive power of the crystalline lens, the eye can produce sharp image at a fixed distance of objects at a varying distance



Accommodation amplitude

- the image distance q is the same
- accommodation amplitude: the difference between the optical power at the near point D_p and that at the far point D_r

$$\Delta D = D_\mathrm{p} - D_\mathrm{r} = \frac{1}{p_\mathrm{p}} + \frac{n}{q} - \left(\frac{1}{p_\mathrm{r}} + \frac{n}{q}\right)$$

$$\Delta D = \frac{1}{p_\mathrm{p}} - \frac{1}{p_\mathrm{r}}$$

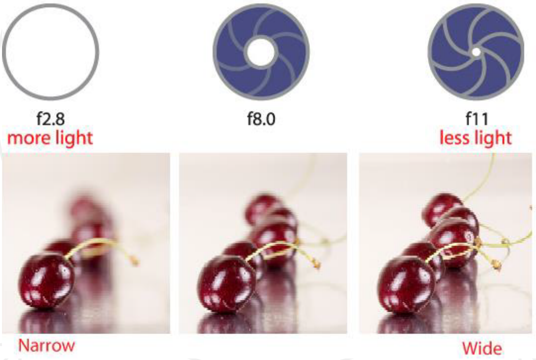
- it declines with age

Vision defects

- short-sightedness: the axis of the eye is too long, sharp image is formed in front of the retina ☛ correction: diverging lens
- long-sightedness: the axis of the eye is too short, sharp image would be formed behind the retina ☛ correction: converging lens
- age of sight: crystalline lens too rigid; the near point is too far away ☛ correction: converging lens (‘reading glass’)
- astigmatism: optical power of the eye is greater in one direction than in the perpendicular direction ☛ correction: cylindrical lens

Adaptation

- the pupil controls the intensity of light reaching the retina (from $10^{-6}\,\mathrm{cd/m^2}$ to $10^{5}\,\mathrm{cd/m^2}$)
- smaller aperture ☛ greater depth of field



Resolution of the eye

- the resolution of the eye is limited by diffraction: the image of a circular dot is a disc surrounded by rings (Airy disc)
- Rayleigh criterion of resolution: two images are just resolvable when the centre of the diffraction disc of one is directly over the first minimum of the other ☛
$\sin\theta = 1.22\frac{\lambda}{d}$

well resolved just resolved not resolved