UNIVERSITAS SCIENTIARUM SZEGEDIENSIS
SZEGEDI TUDOMÁNYEGYETEM

Medical Physics 3.
Physics of muscles

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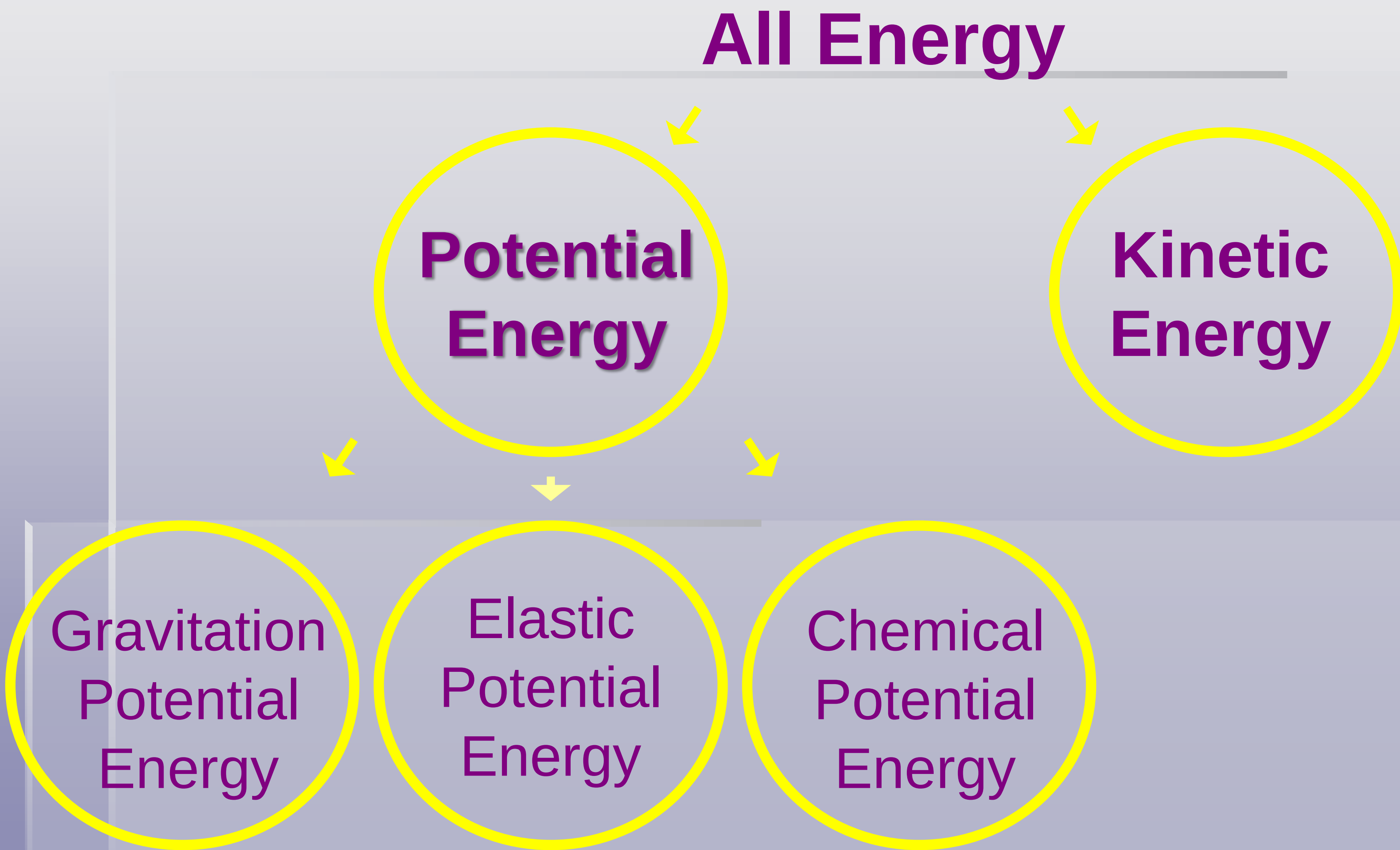
Szeged, 21th September 2017

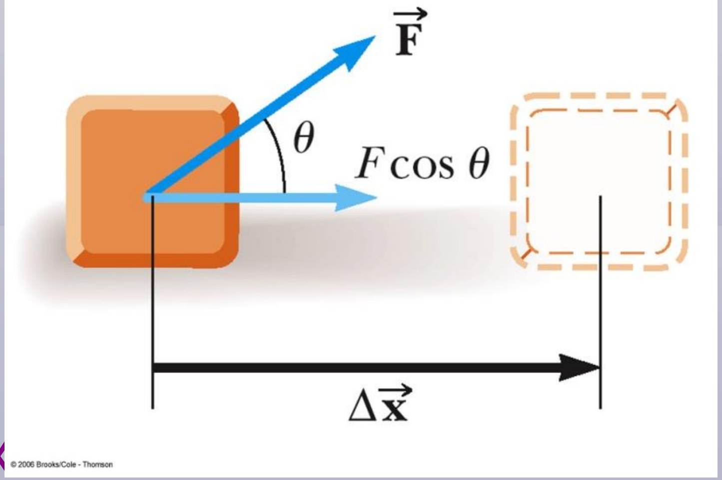
How is all energy divided?



Definition of Work *W*

- The work, *W*, done by a constant force on an object is defined as the product of the component of the force along the direction of displacement and the magnitude of the displacement

- W=F x cosϴ x Δx
  - F is the magnitude of the force
  - Δ x is the magnitude of the object's displacement
  - q is the angle between F and Δx



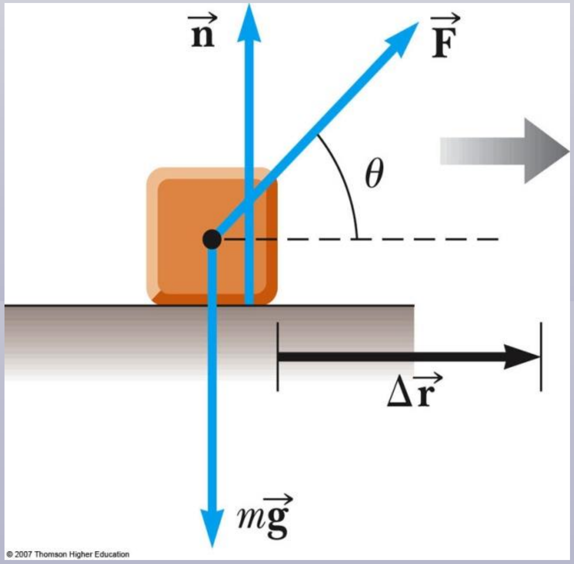
Work Done by Multiple Forces

- If more than one force acts on an object, then the total work is equal to the algebraic sum of the work done by the individual forces

$$W_{\text{net}} = \sum W_{\text{by individual forces}}$$

  - Remember work is a scalar, so this is the algebraic sum

$$W_{net} = W_g + W_N + W_F = (F\cos\theta)\Delta r$$



Kinetic Energy and Work

- Kinetic energy associated with the motion of an object

$$KE = \frac{1}{2}mv^2$$

- Scalar quantity with the same unit as work
- Work is related to kinetic energy

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\ 2} = (F_{net}\cos\theta)\Delta x$$
$$= \int_{x_i}^{x_f} \mathbf{F} \cdot d\mathbf{r}$$

Units: N-m or J

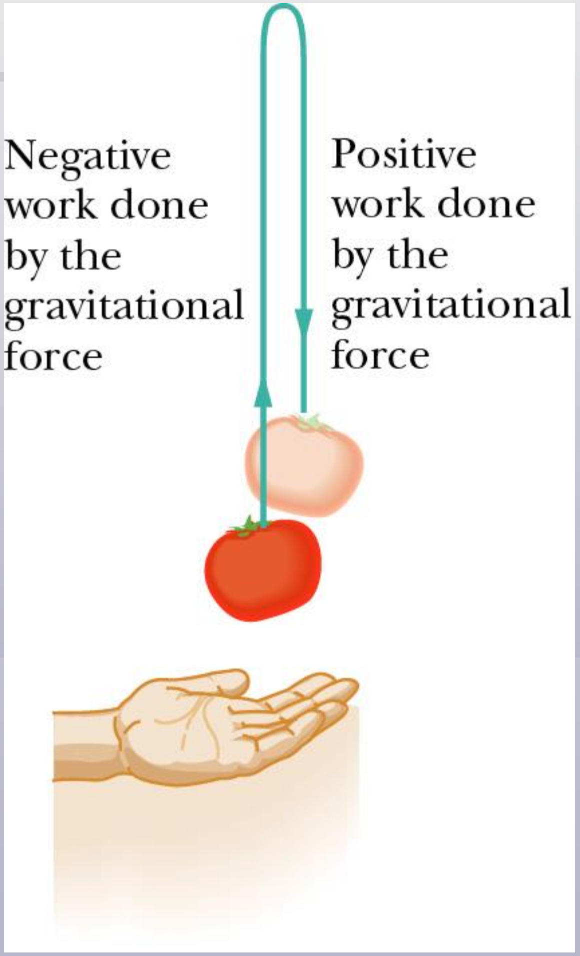
$$W_{net} = KE_f - KE_i = \Delta KE$$

Work done by a Gravitational Force

- Gravitational Force
  - Magnitude: mg
  - Direction: downwards to the Earth's center
- Work done by Gravitational Force

$$W = F\Delta r \cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

$$W_g = mg\Delta r \cos\theta$$

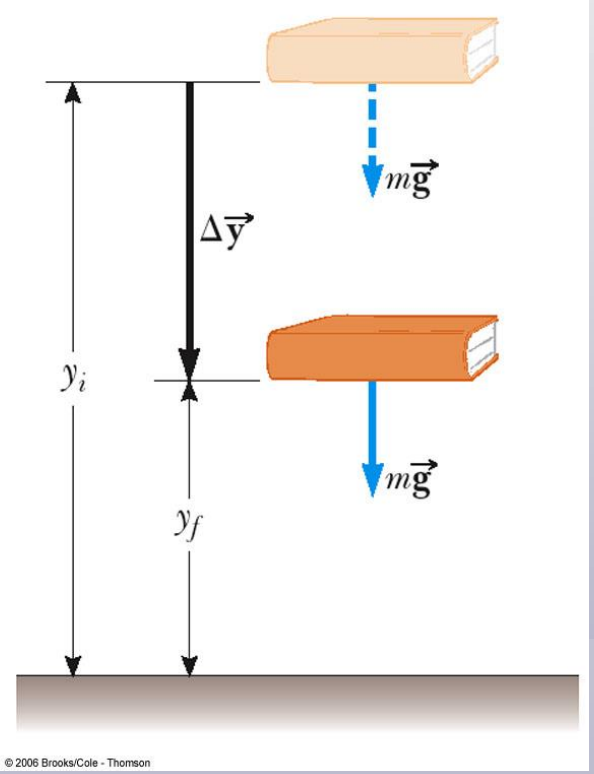


Potential Energy

- Potential energy is associated with the position of the object
- Gravitational Potential Energy is the energy associated with the relative position of an object in space near the Earth's surface
- The gravitational potential energy

$$PE \equiv mgy$$

  - m is the mass of an object
  - g is the acceleration of gravity
  - y is the vertical position of the mass relative the surface of the Earth
  - SI unit: joule (J)

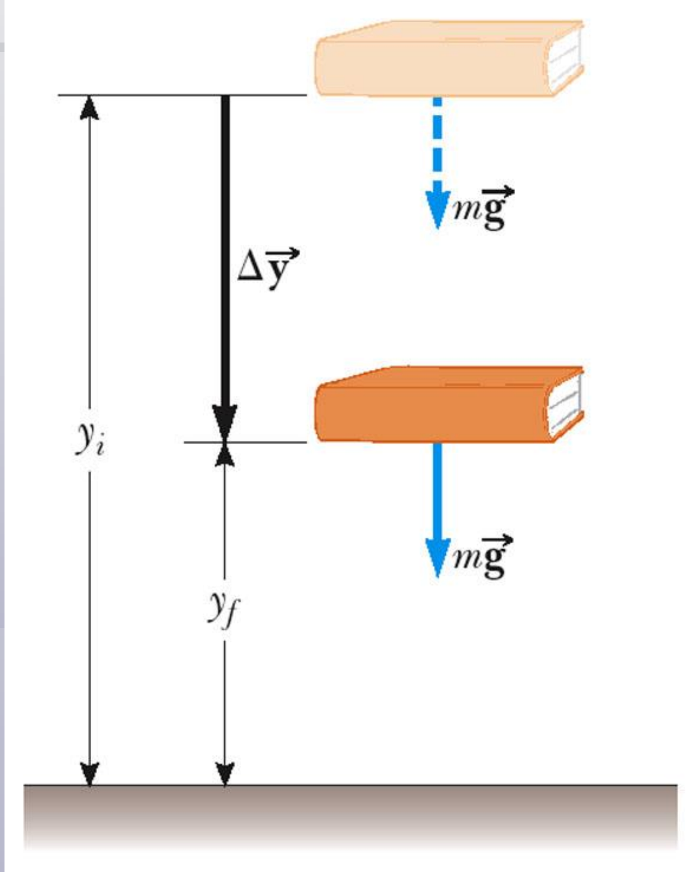


Reference Levels

- A location where the gravitational potential energy is zero must be chosen for each problem
  - The choice is arbitrary since the change in the potential energy is the important quantity
  - Choose a convenient location for the zero reference height
    - often the Earth's surface
    - may be some other point suggested by the problem
  - Once the position is chosen, it must remain fixed for the entire problem

Work and Gravitational Potential Energy

- $PE = mgy$
- $$W_g = F\Delta y\cos\theta = mg(y_f - y_i)\cos 180$$
  $$= -mg(y_f - y_i) = PE_i - PE_f$$
- Units of Potential Energy are the same as those of Work and Kinetic Energy



$W_{gravity} = \Delta KE = -\Delta PE = PE_i - PE_f$

Extended Work-Energy Theorem

- The work-energy theorem can be extended to include potential energy:

$$W_{net} = KE_f - KE_i = \Delta KE$$

$$W_{gravity} = PE_i - PE_f$$

- If we only have gravitational force, then $W_{net} = W_{gravity}$

$$KE_f - KE_i = PE_i - PE_f$$

$$KE_f + PE_f = PE_i + KE_i$$

- The sum of the kinetic energy and the gravitational potential energy remains constant at all time and hence is a conserved quantity

Extended Work-Energy Theorem

- We denote the total mechanical energy by

$$E = KE + PE$$

- Since $$KE_f + PE_f = PE_i + KE_i$$

- The total mechanical energy is conserved and remains the same at all times

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

Instantaneous Power

- Power is the time rate of energy transfer. Power is valid for any means of energy transfer
- Other expression

$$\overline{P} = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\overline{v}$$

- A more general definition of instantaneous power

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

Units of Power

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second = 1 kg · m^2 / s^3
- Units of power can also be used to express units of work or energy
  - 1 kWh = (1000 W)(3600 s) = 3.6 x10^6 J

Conservative forces

Kinetic energy ⟺ Velocity

Potential energy?

It is defined only for a certain class of forces called conservative forces.

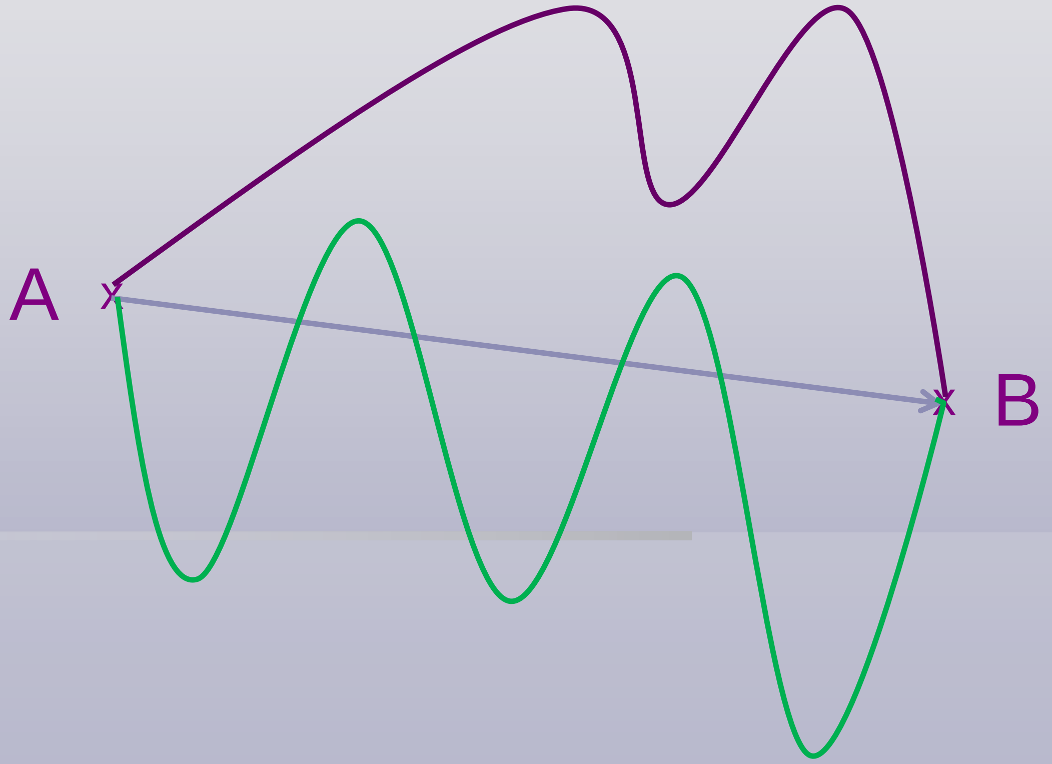
What are conservative forces?

Do *spring force*, *gravitational force*, and *frictional force et al.* belong to conservative forces?

A force is conservative if the work it does on an object moving between two points is independent of the path the objects take between the points!

The work depends only upon the initial and final positions of the object

Any conservative force can have a potential energy function associated with it



Work done by gravity

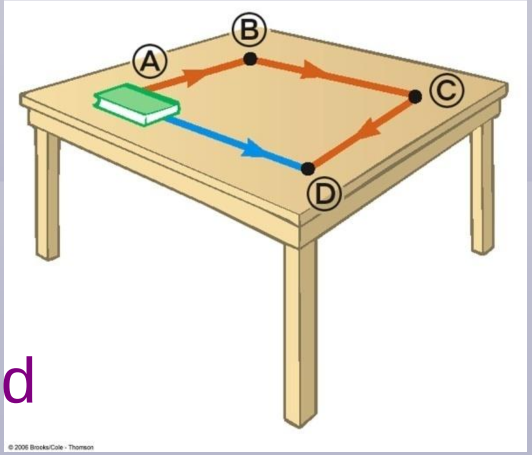
Work done by spring force

Nonconservative Forces

- A force is nonconservative if the work it does on an object depends on the path taken by the object between its final and starting points.
  - The work depends upon the movement path
  - For a non-conservative force, potential energy can NOT be defined
  - Work done by a nonconservative force

$$W_{nc} = \sum \vec{F} \cdot \vec{d} = -f_k d + \sum W_{otherforces}$$

  - It is generally dissipative. The dispersal of energy takes the form of heat or sound

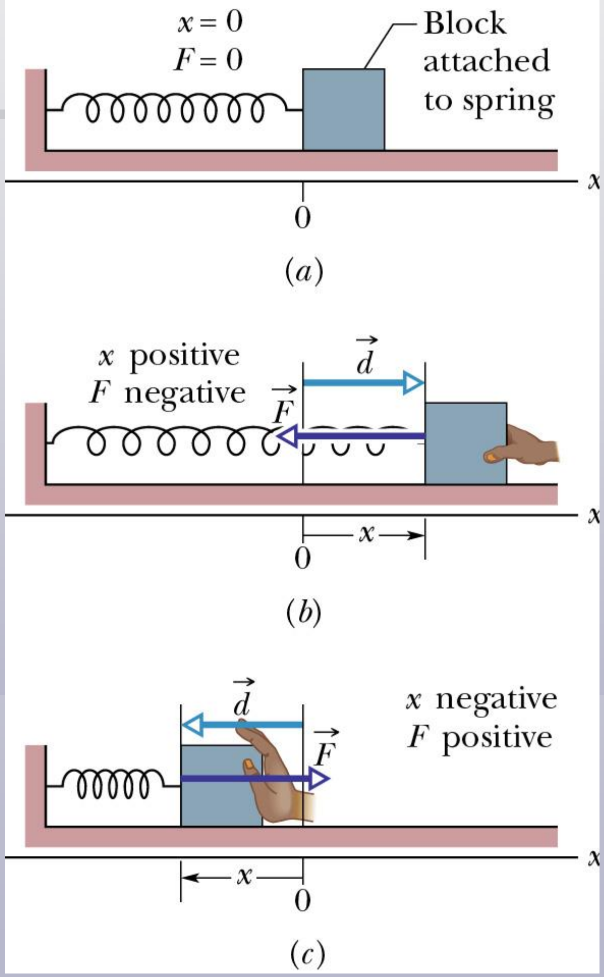


Spring Force

- Involves the *spring constant*, k
- Hooke's Law gives the force

$$\vec{F} = -k\vec{d}$$

  - F is in the opposite direction of displacement d, always back towards the equilibrium point.
  - k depends on how the spring was formed, the material it is made from, thickness of the wire, etc. Unit: N/m.



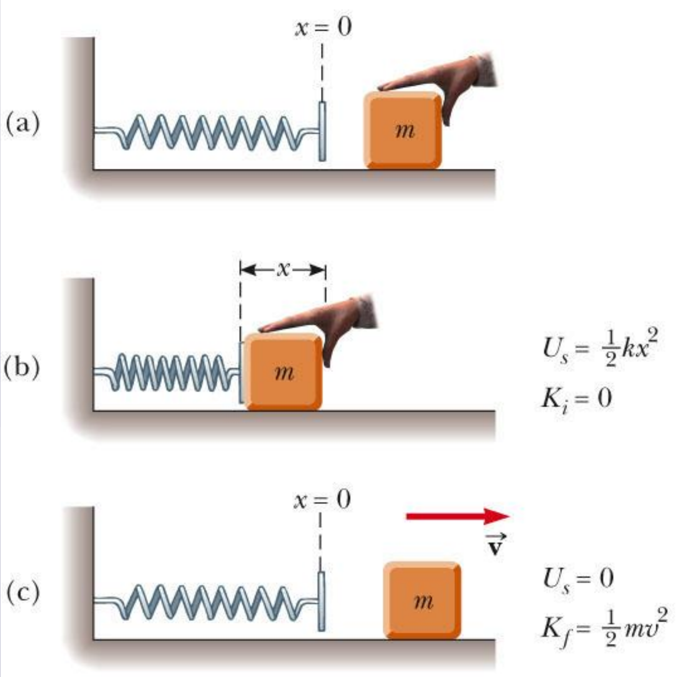
Potential Energy in a Spring

$$PE_s = \frac{1}{2}kx^2$$

- Elastic Potential Energy:
  - SI unit: Joule (J)
  - related to the work required to compress a spring from its equilibrium position to some final, arbitrary, position x
- Work done by the spring

$$W_s = \int_{x_i}^{x_f} (-kx)dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_s = PE_{si} - PE_{sf}$$



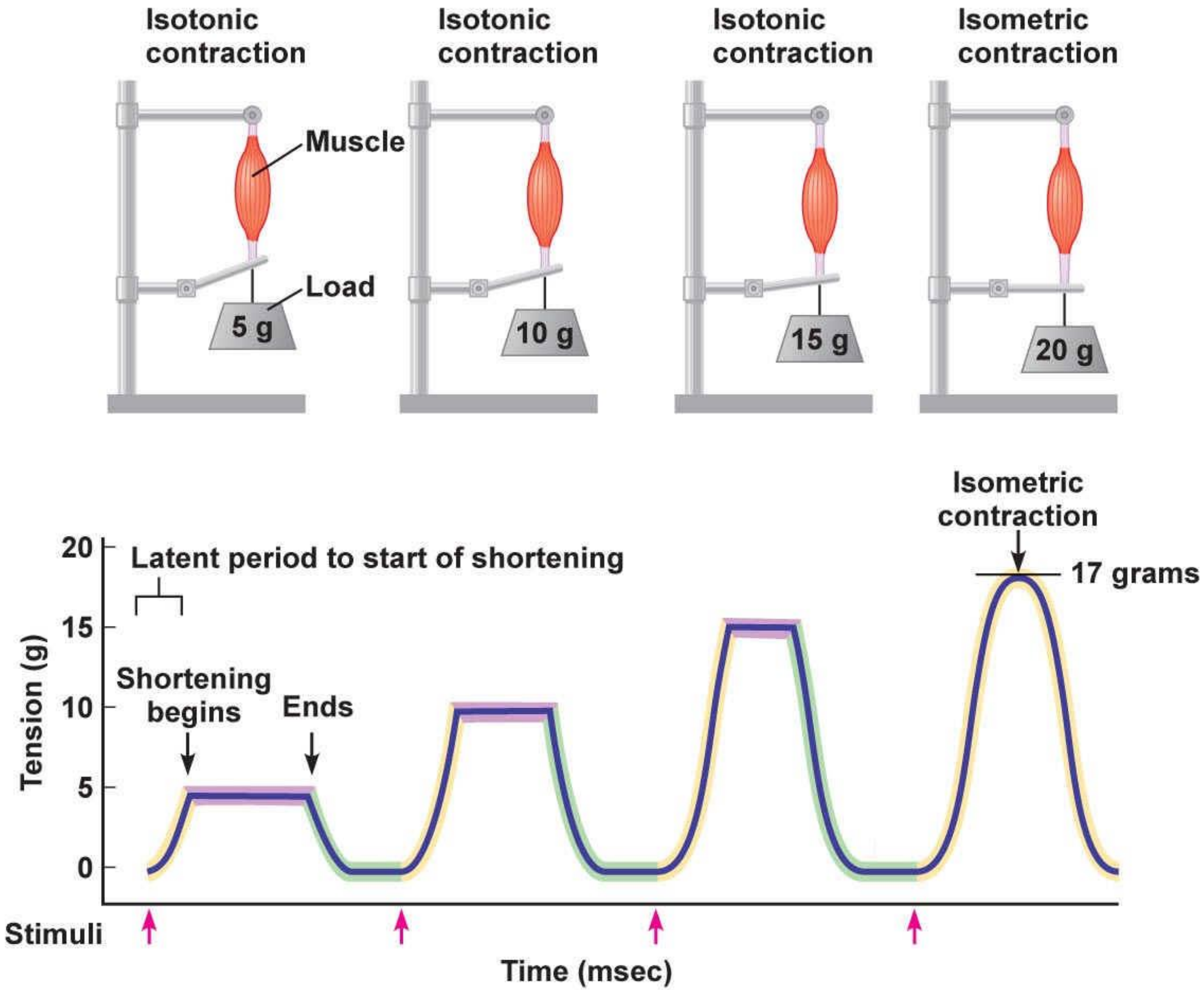
Energy Review

- Kinetic Energy
  - Associated with movement of members of a system
- Potential Energy
  - Determined by the configuration of the system
  - Gravitational and Elastic
- Internal Energy
  - Related to the temperature of the system

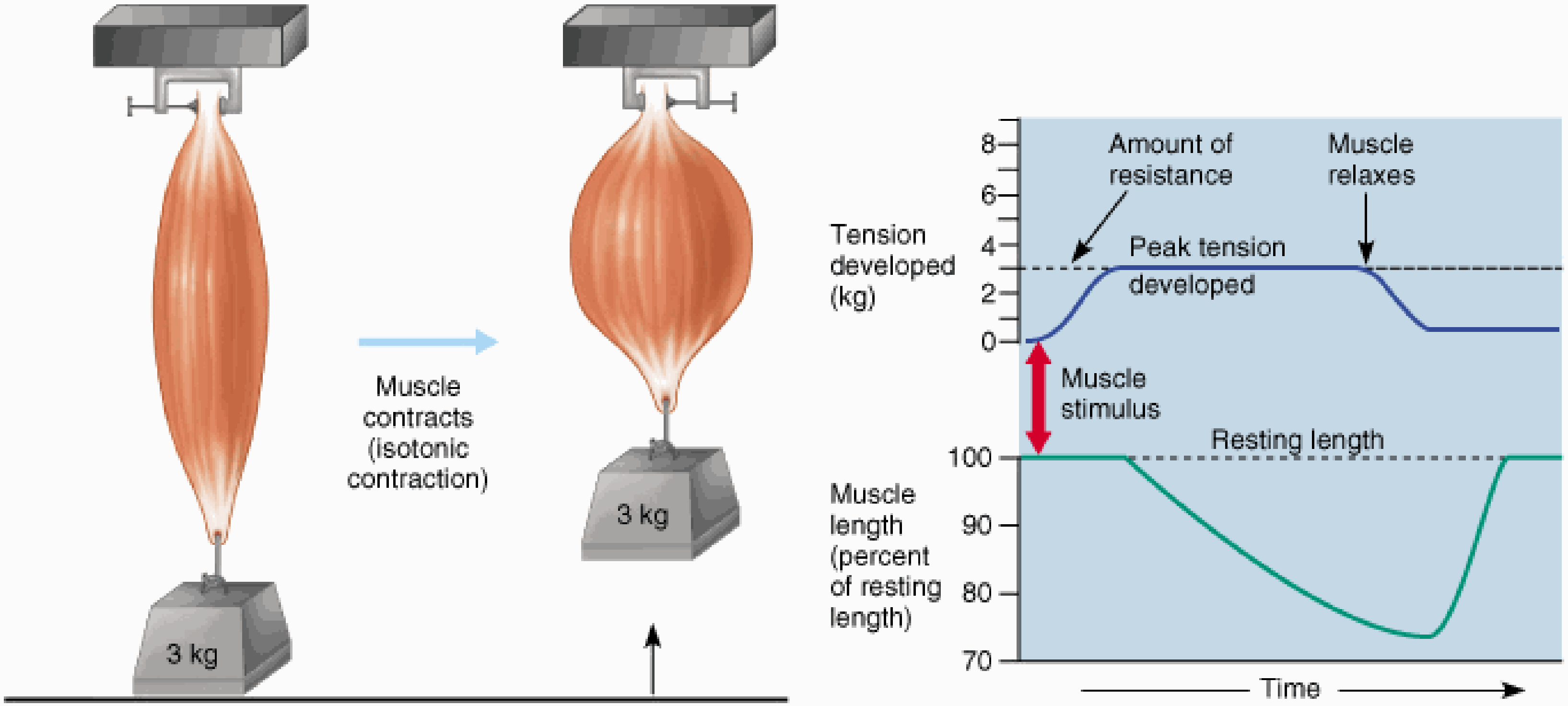
- Muscle types:
  - cardiac muscle: composes the heart
  - smooth muscle: lines hollow internal organs
  - skeletal (striated or voluntary) muscle: attached to skeleton via tendon & movement
- Skeletal muscle 40-45% of body weight
  - > 430 muscles
  - ~ 80 pairs produce vigorous movement
- Dynamic & static work
  - Dynamic: locomotion & positioning of segments
  - Static: maintains body posture

III. Force production in muscle

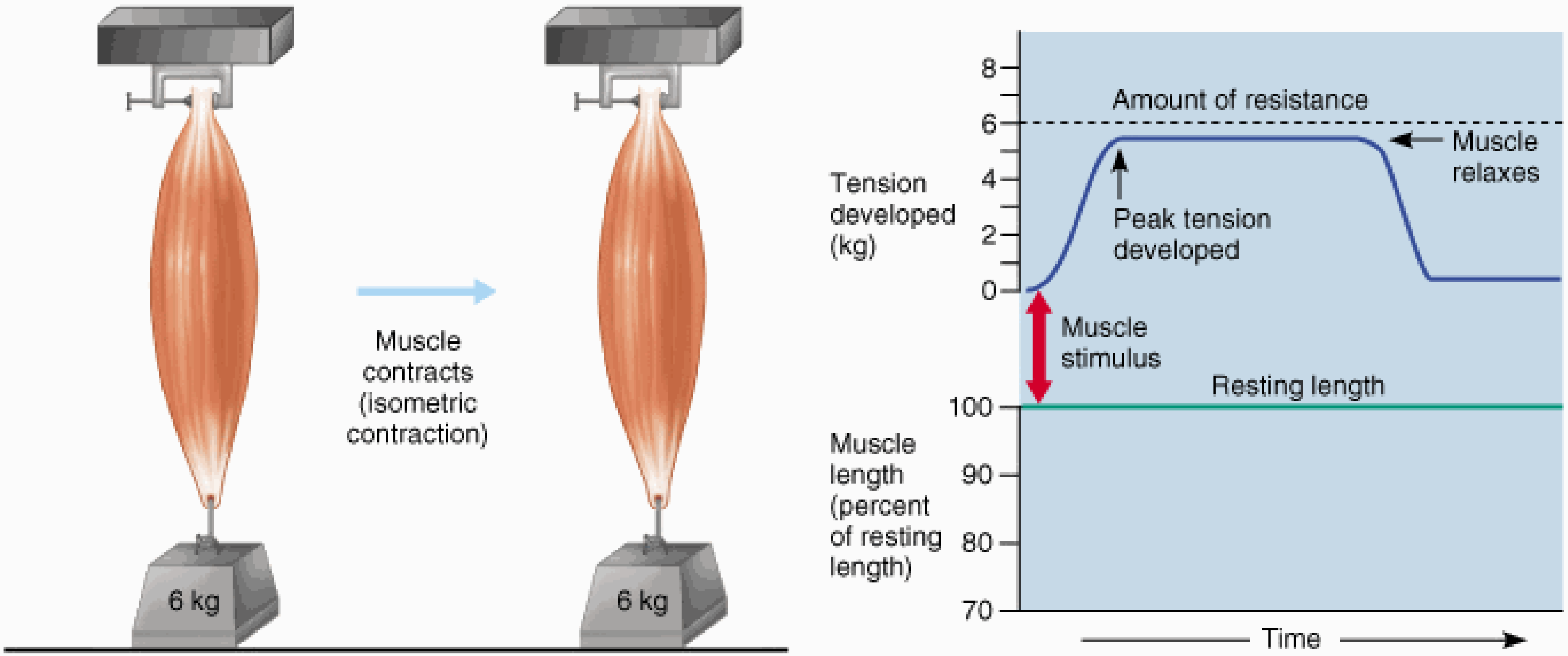
- Force –length characteristics
- Force – velocity characteristics
- Muscle Modeling
- Neuromuscular system dynamics


Isotonic contraction
Isotonic contraction
Isotonic contraction
Isometric contraction
Muscle
Load
5 g
10 g
15 g
20 g
Isometric contraction
17 grams
20
15
10
5
0
Latent period to start of shortening
Shortening begins
Ends
Tension (g)
Stimuli
Time (msec)

Isotonic contraction

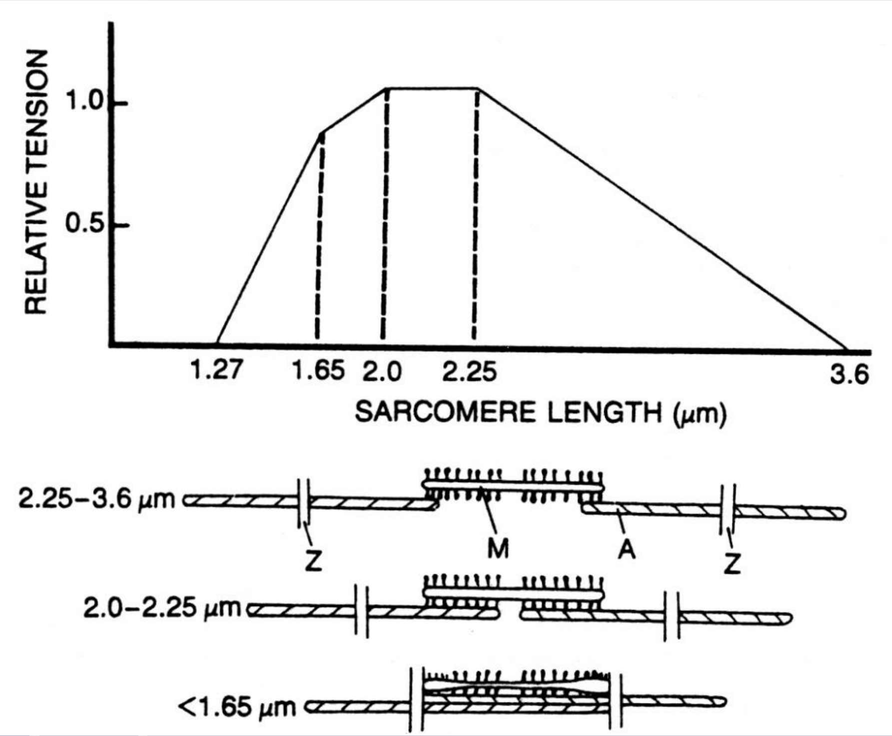


(a) Isotonic (concentric) contraction



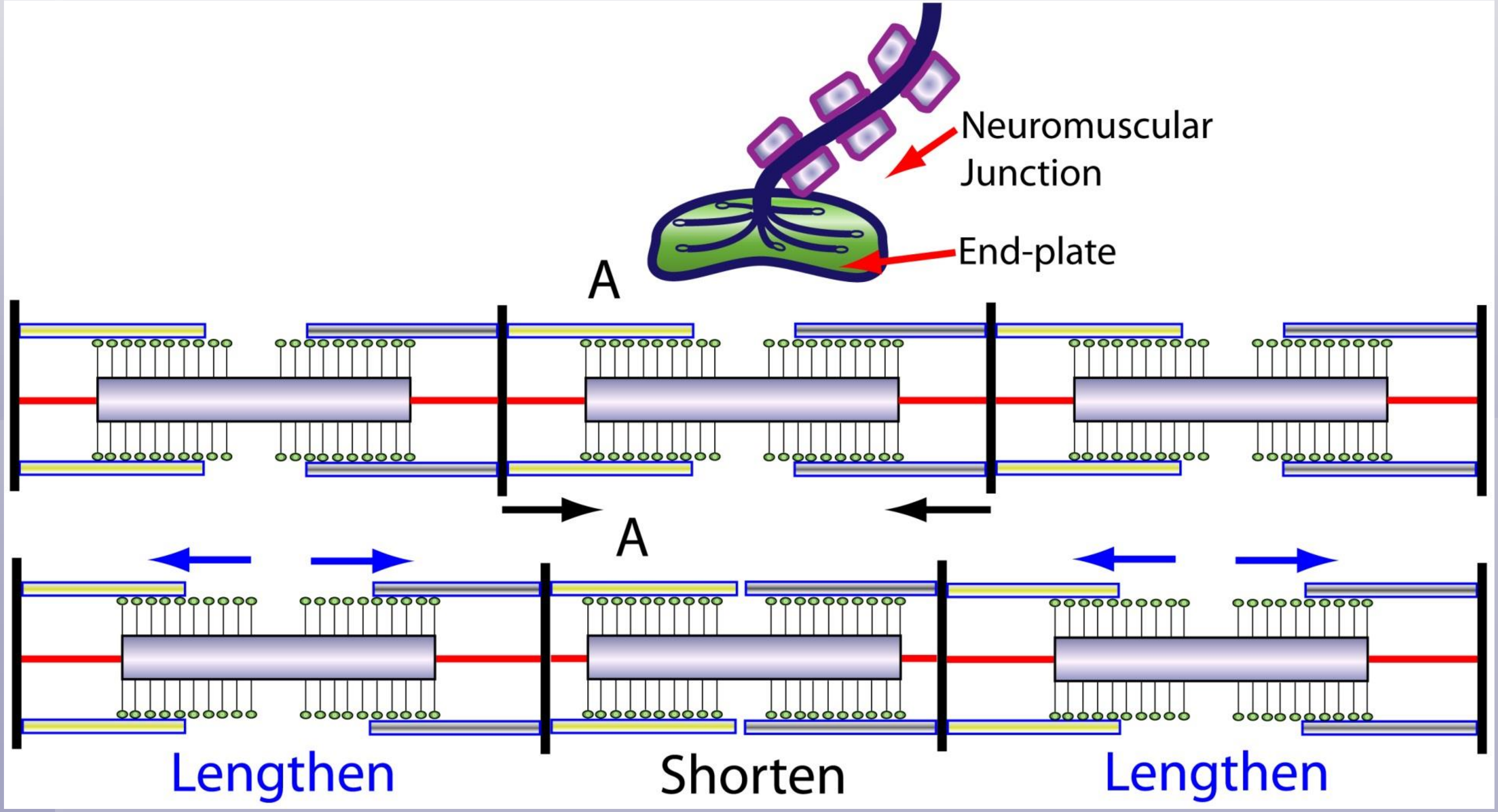
(b) Isometric contraction

3-1 Force-length curve of contractile component

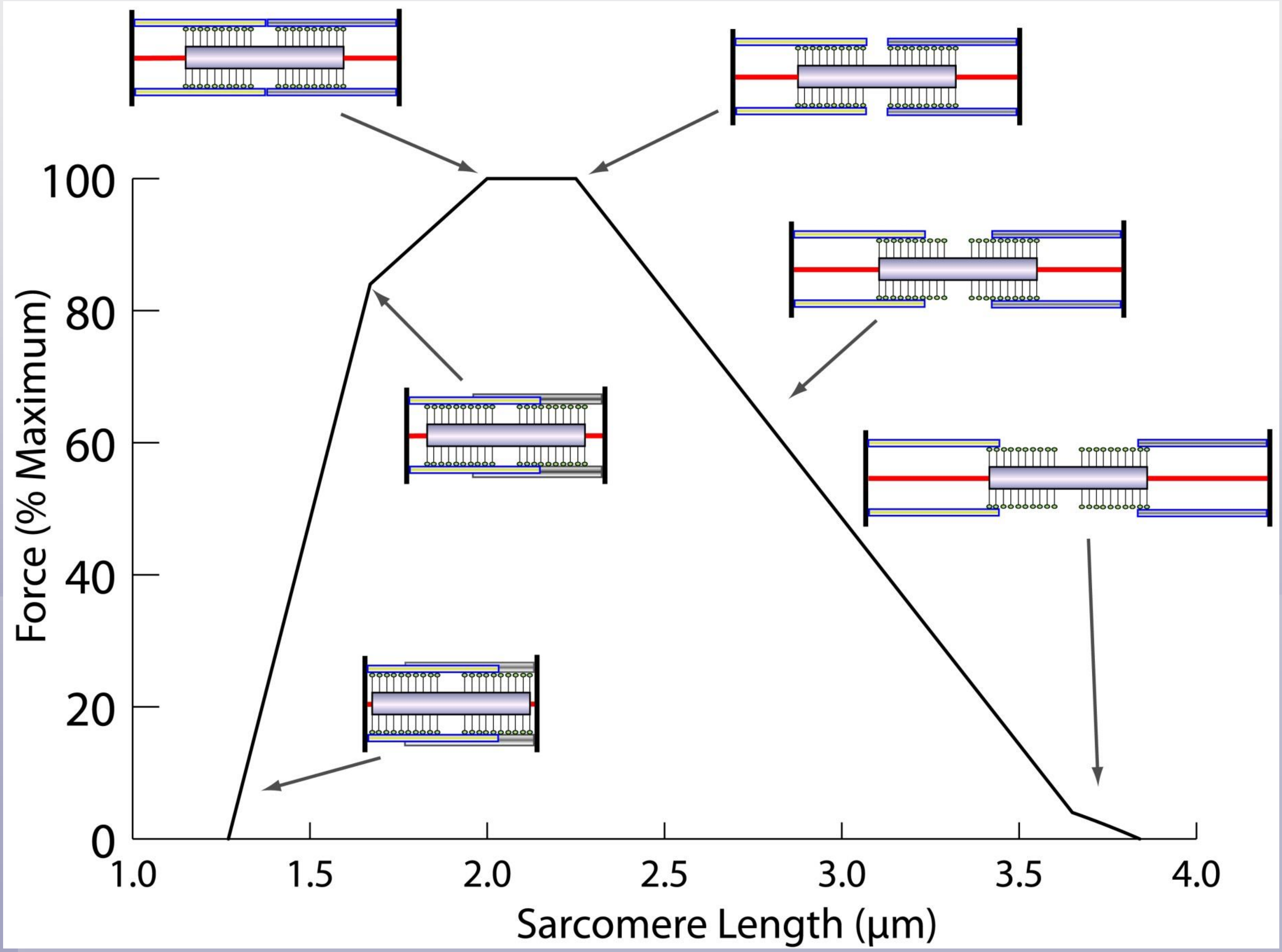


- Resting 2.0-2.25 um max. no. of cross bridges; max. tension
- 2.25-3.6 um no. of cross bridge ↓
- < 1.65 um overlap of actin no. of cross bridge ↓

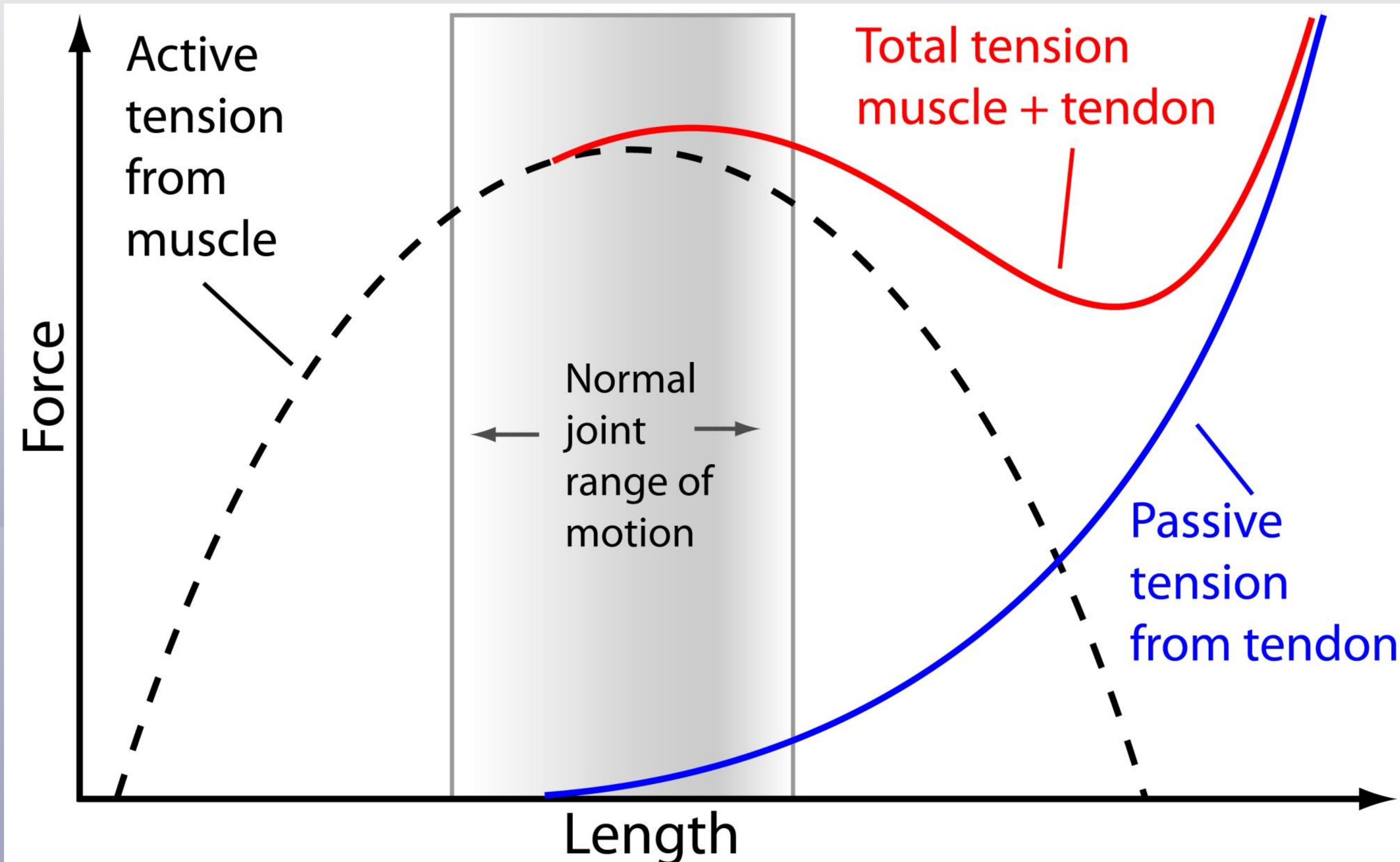
Length changes to an individual sarcomere during an isometric contraction. The sarcomere directly underneath the end-plate will be the first to develop tension which causes the sarcomeres to the right and left to lengthen.



Sarcomere Force – Length Relationship

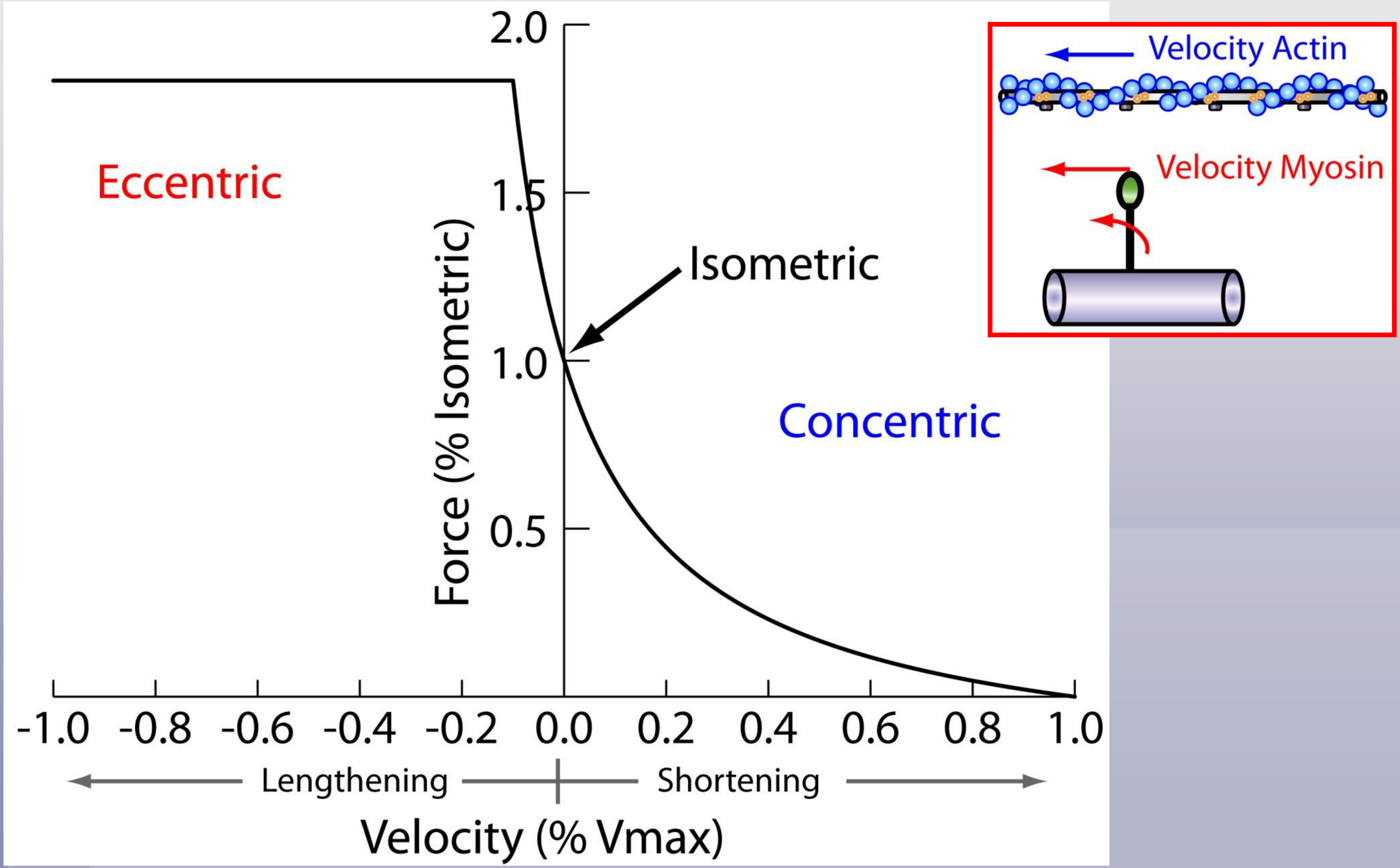


Force – Length in Normal Joint Range of Motion

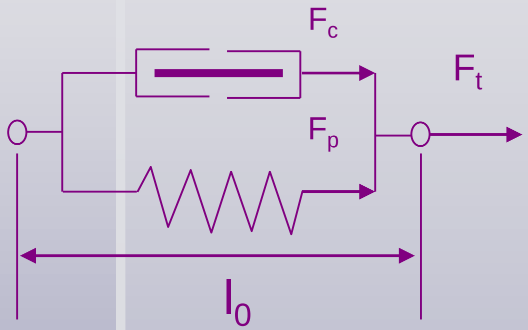


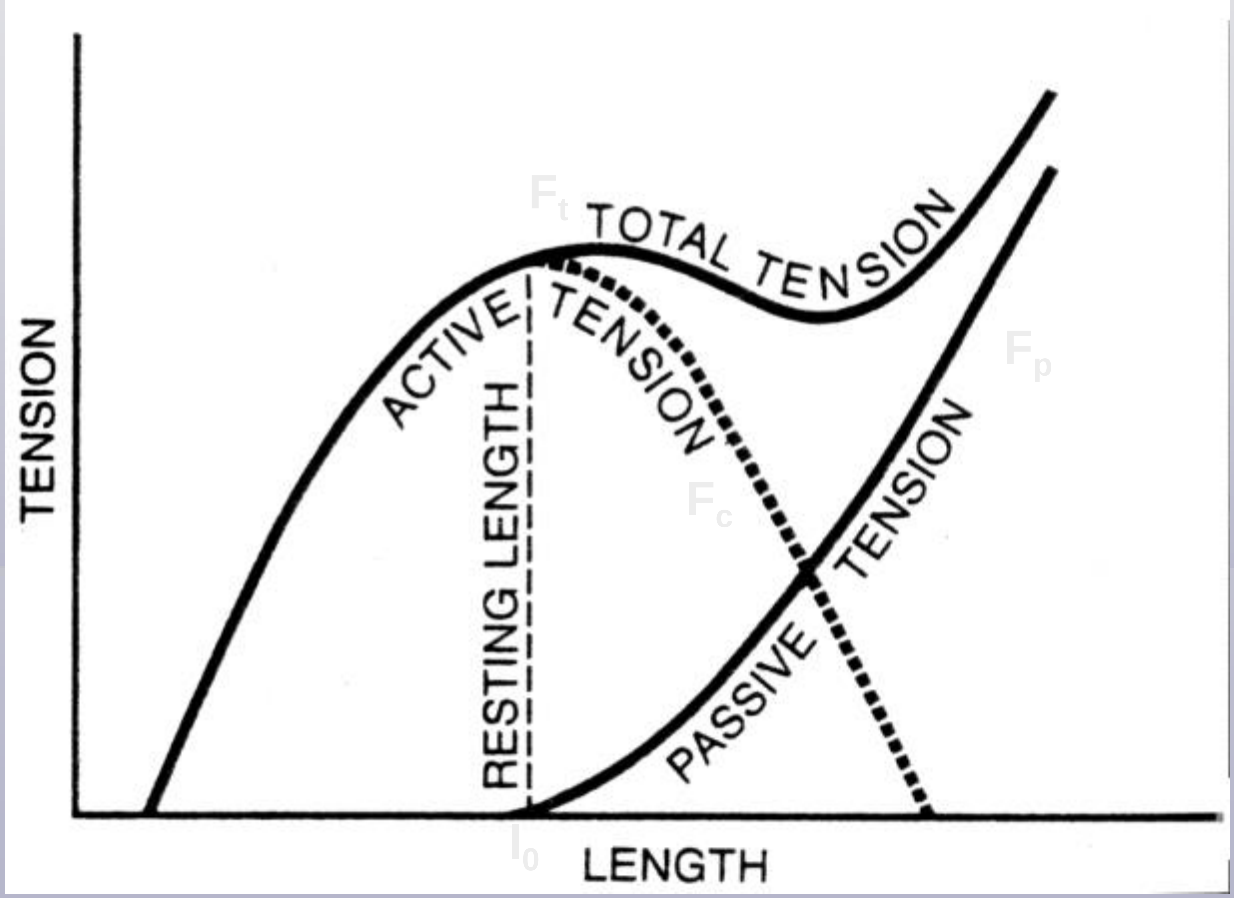
Force – Velocity Relationship

$$(P + a) V = b (Po - P)$$

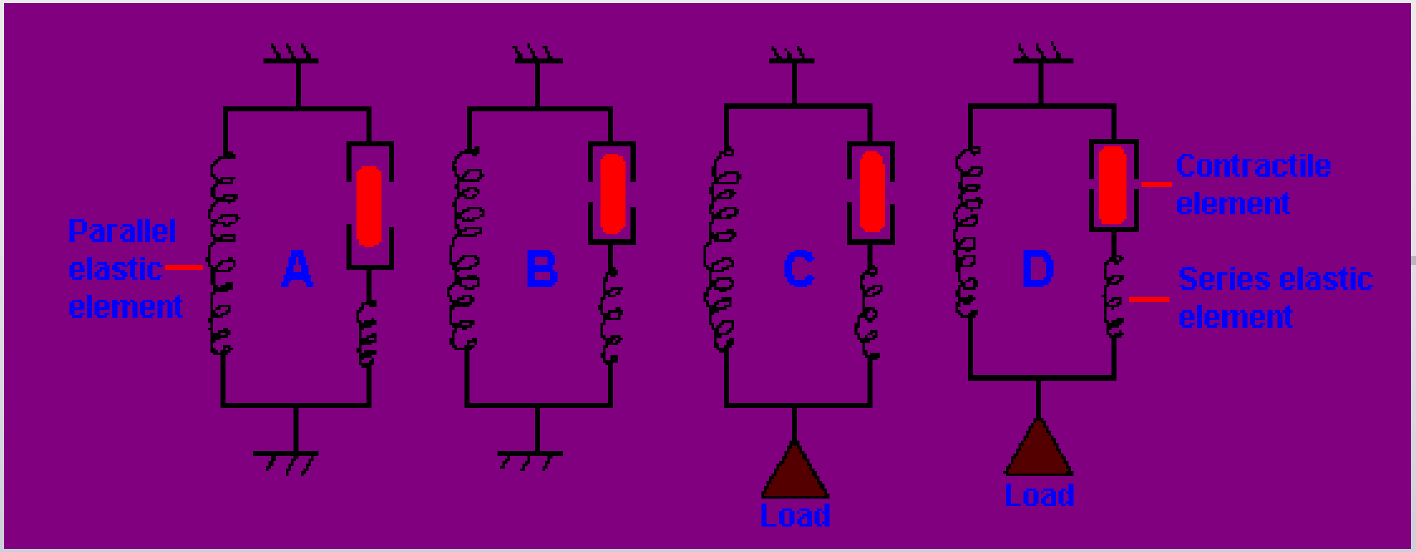


Influence of parallel elastic component





Note: F_c is under voluntary control & F_p is always present



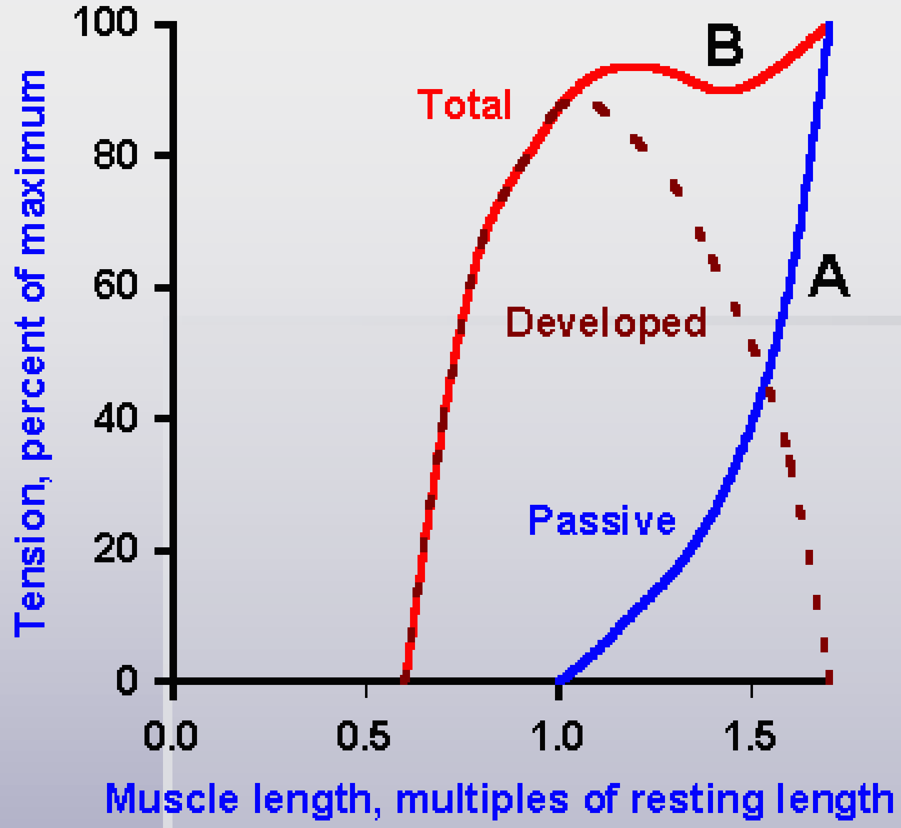
Series and parallel elastic elements in muscle.

A. Resting muscle contains elastic elements in series with the contractile elements (sarcomeres) and in parallel with them.

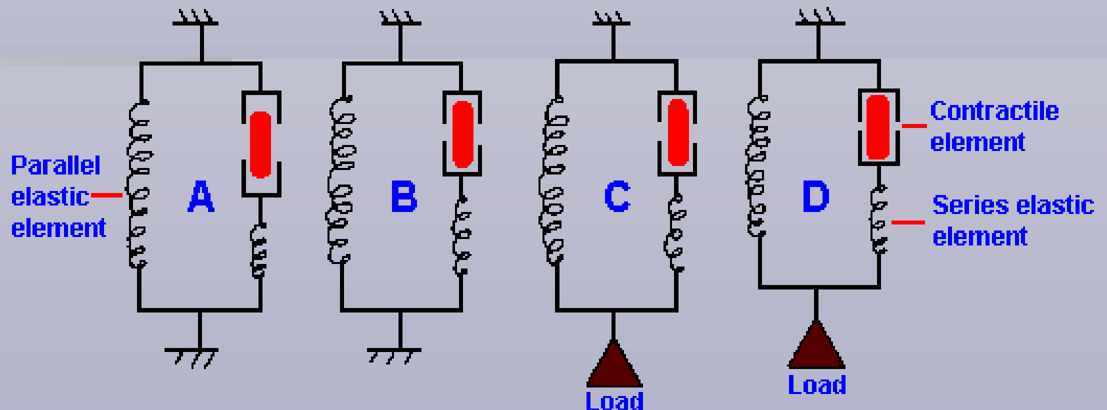
B. During an isometric contraction, the muscle does not change length, but sarcomeres shorten, stretching the series elastic elements.

C. During isotonic contraction, the contractile elements shorten, stretching the series elastic elements, before they develop tension to lift the load.

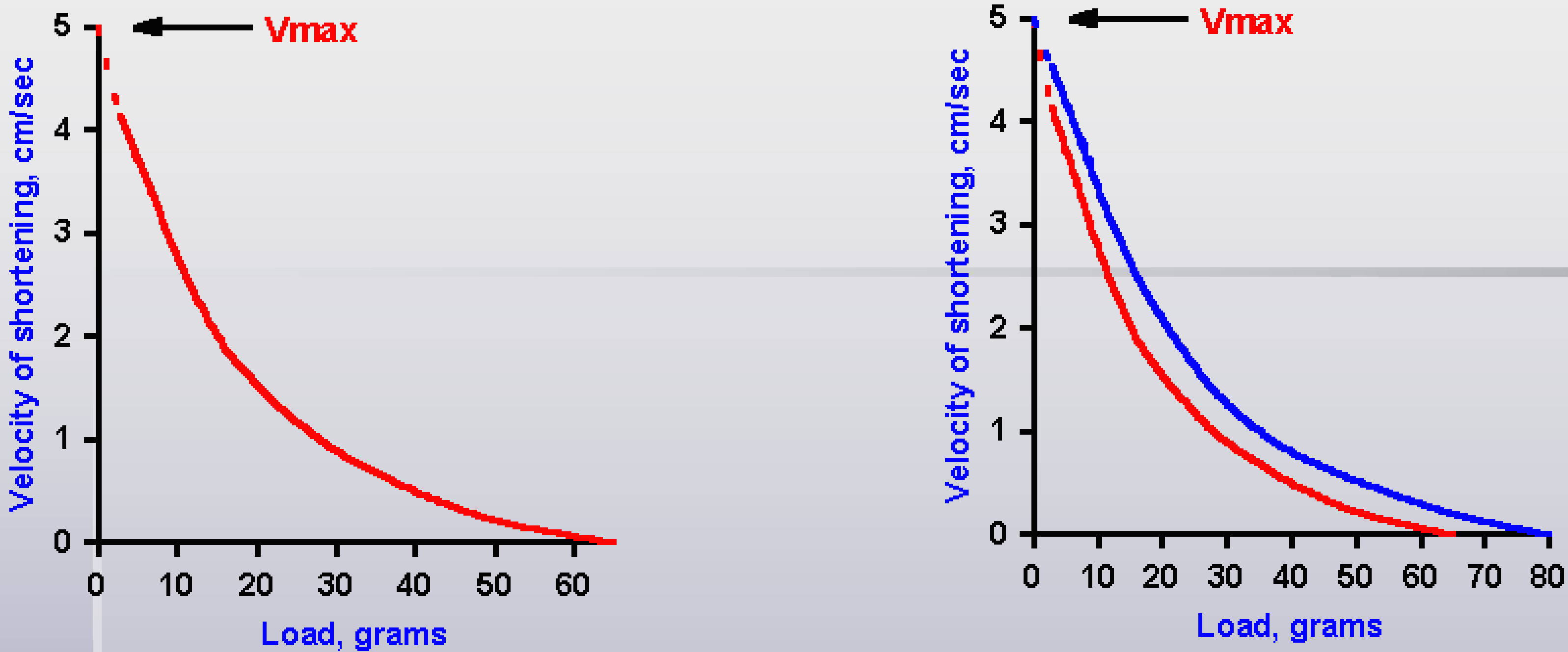
D. Muscle begins to shorten when contractile elements shorten further.



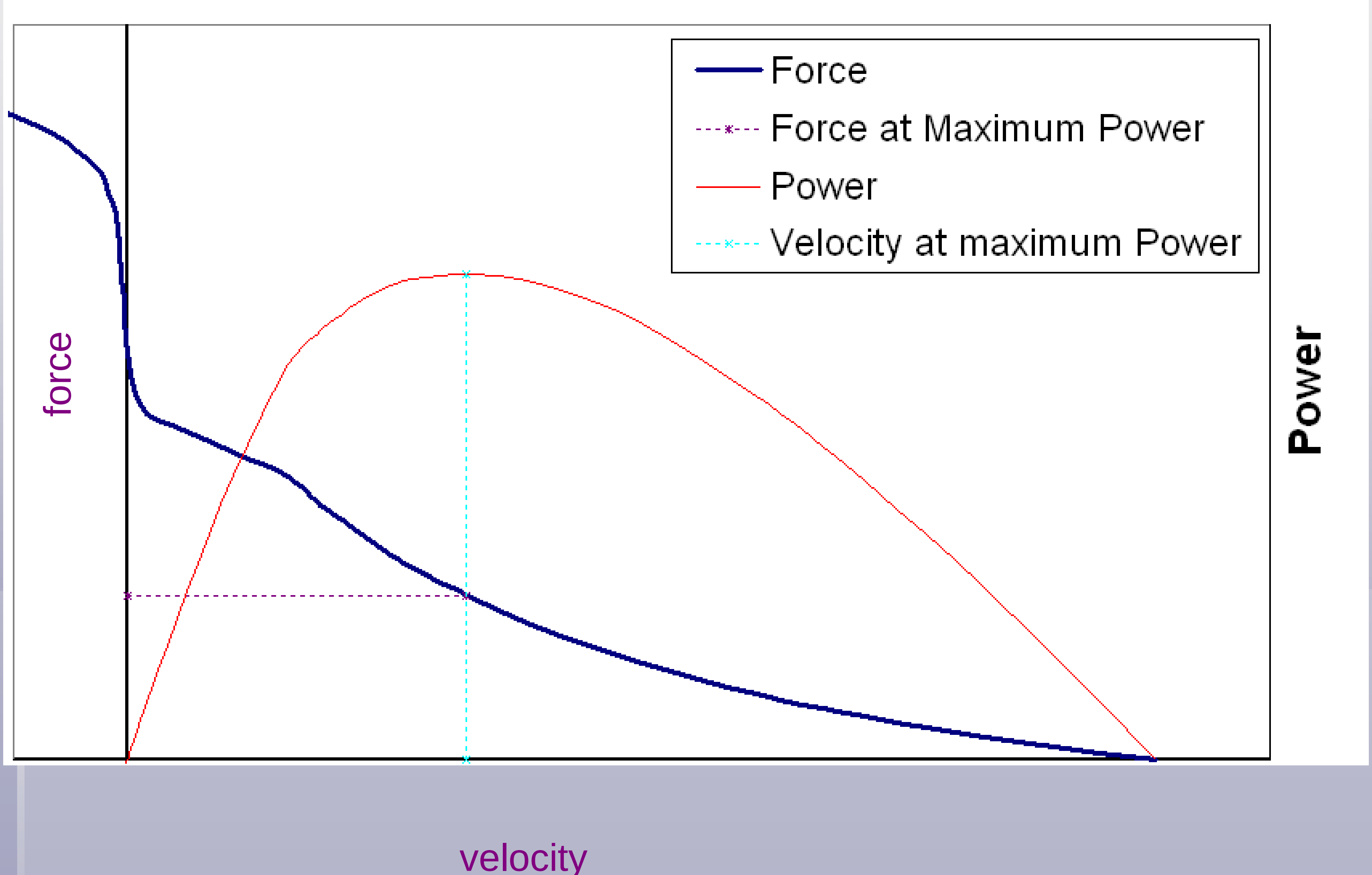
100
80
60
40
20
0
0.0
0.5
1.0
1.5
Tension, percent of maximum
Muscle length, multiples of resting length
Total
B
Developed
A
Passive

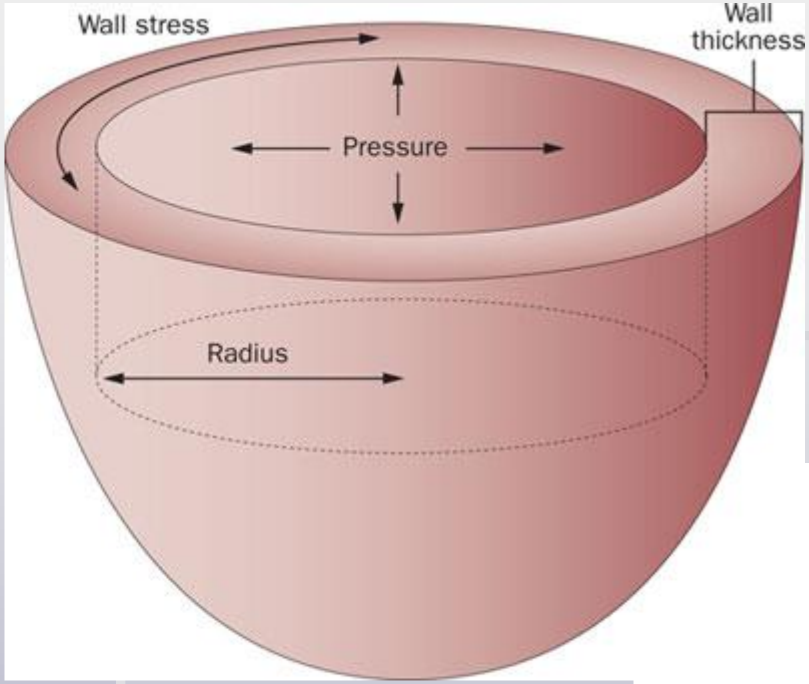


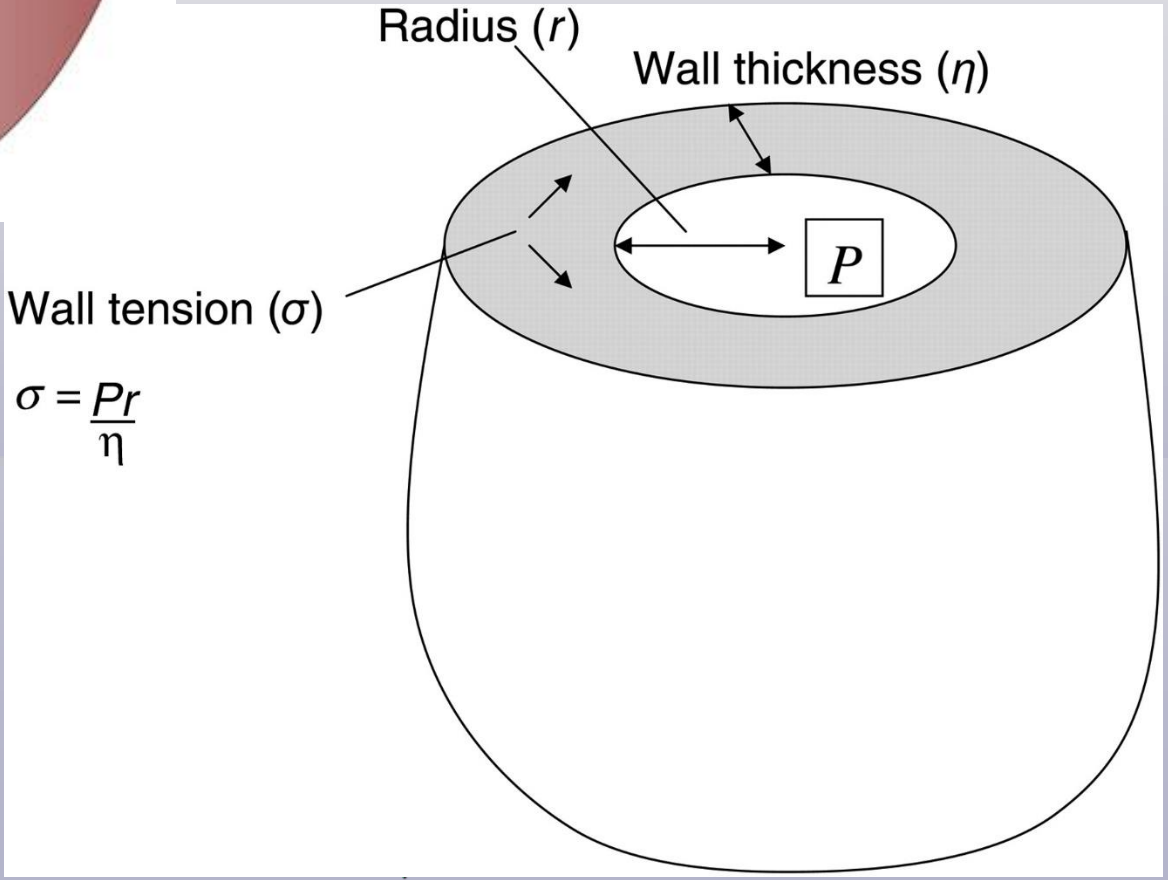
Parallel
elastic
element
A
B
C
D
Contractile
element
Series elastic
element
Load
Load

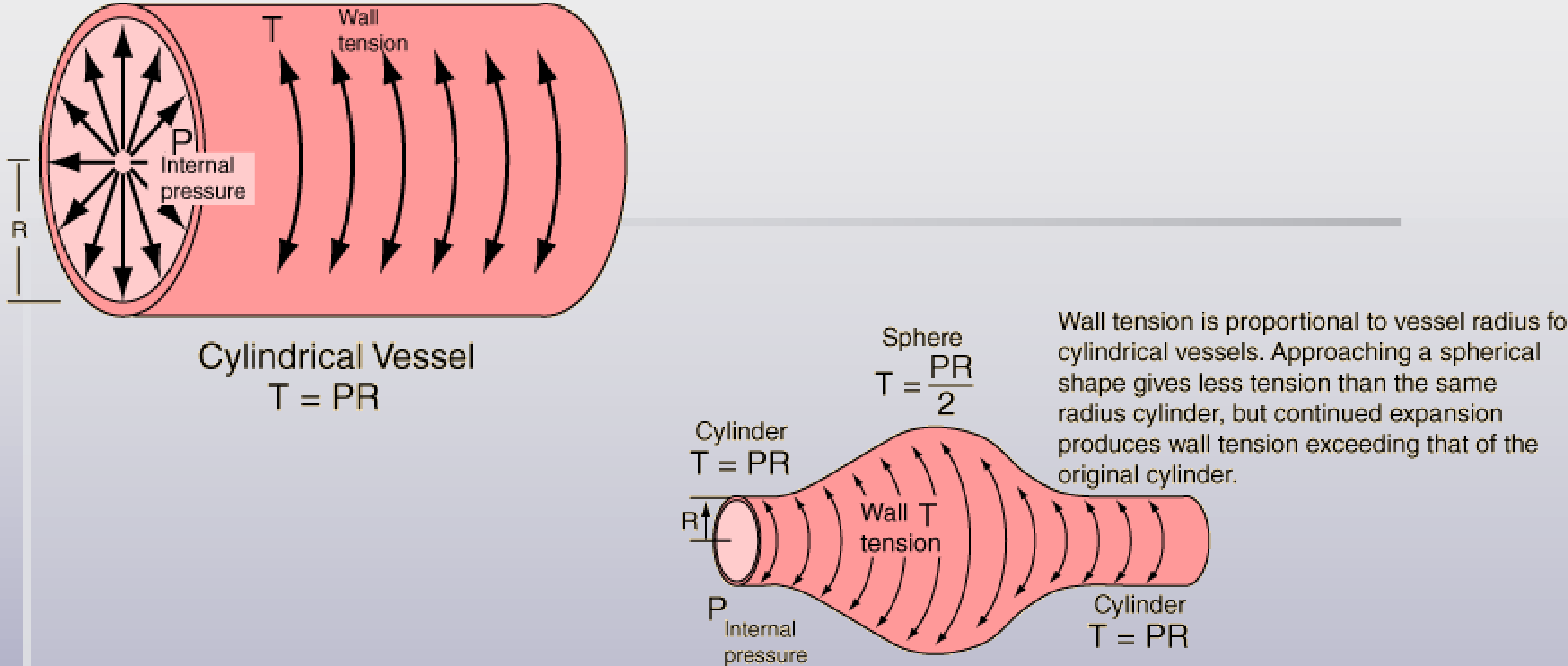


Force-velocity curve. Velocity of shortening (ordinate) is plotted against the load (force) applied to the muscle (abscissa). As the load increases, the velocity of shortening decreases. The curve is extrapolated back to zero load, yielding the maximum velocity the muscle can achieve, V_{max}. (Aidley DJ: *The Physiology of Excitable Cells*. Cambridge, Cambridge Univ Press, 1971)


Force
Force at Maximum Power
Power
Velocity at maximum Power
force
Power
velocity


Wall stress
Wall thickness
Pressure
Radius


Radius (r)
Wall thickness (η)
P
Wall tension (σ)
σ = Pr/η



The tension in the walls of arteries and veins in the human body is a classic example of LaPlace's law. This geometrical law applied to a tube or pipe says that for a given internal fluid pressure, the wall tension will be proportional to the radius of the vessel.

Mechanical work and power of heart

Mechanical power of heart

(for pulse rate 70 min^{-1}) .... 1,3 W

Total power of heart

(at rest conditions) ....13 W

Total power of human organism

(at rest conditions) ....115 W

Mechanical work of heart muscle:

Some very small work dW is done agains external pressure p during ejection of very small blood volume dV:

$dW = p.dV$

The whole work during a systole:

$W = \int p.dV$

Very small part of this work is transformed into kinetic energy of blood ejected.

Estimation of heart work during one systole

- *p = const.* $\Rightarrow$ $W = p_m.\Delta V,$

p_m is mean blood pressure at the beginning of aorta

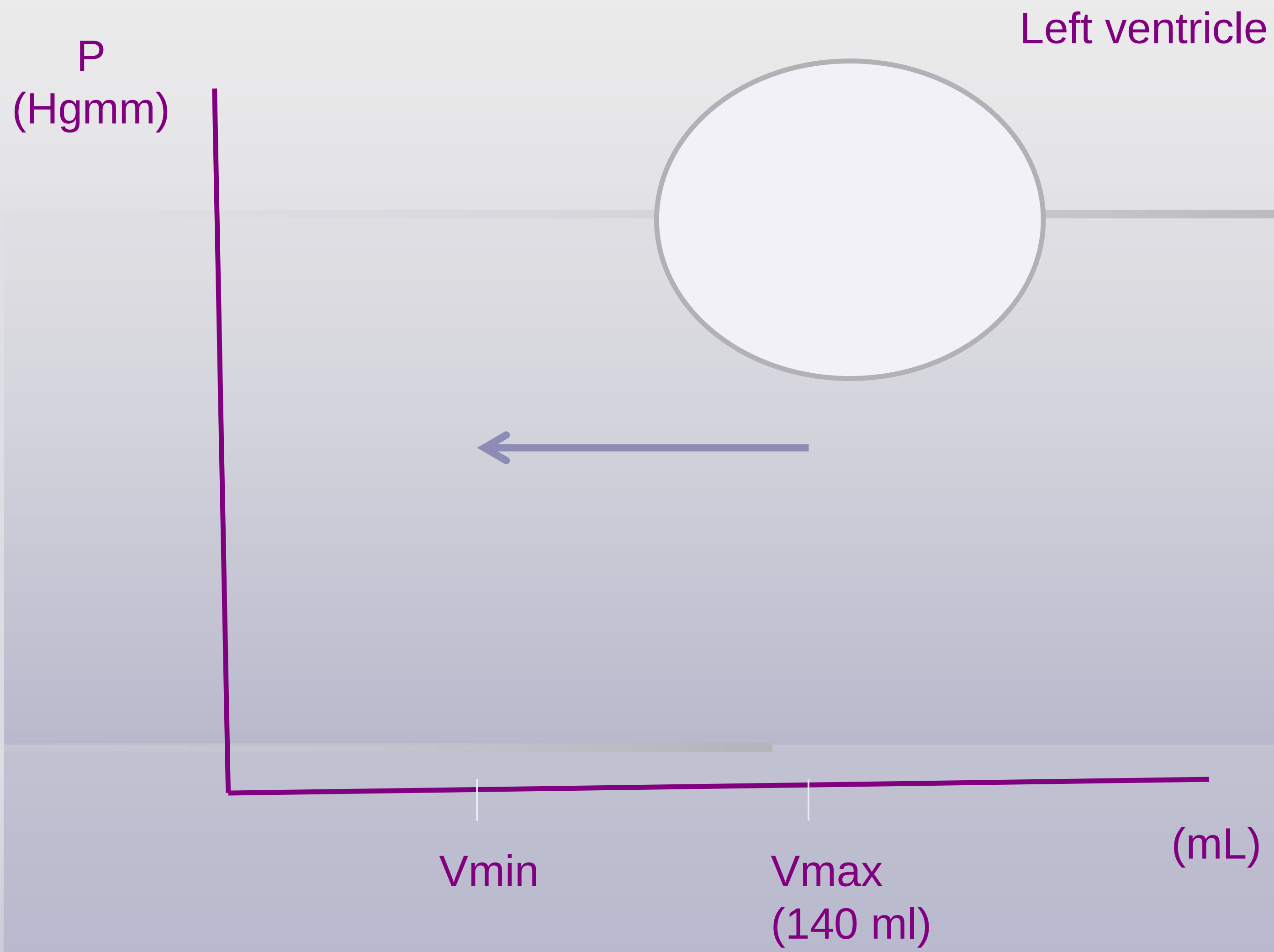
- *Left ventricle* — *Right ventricle*

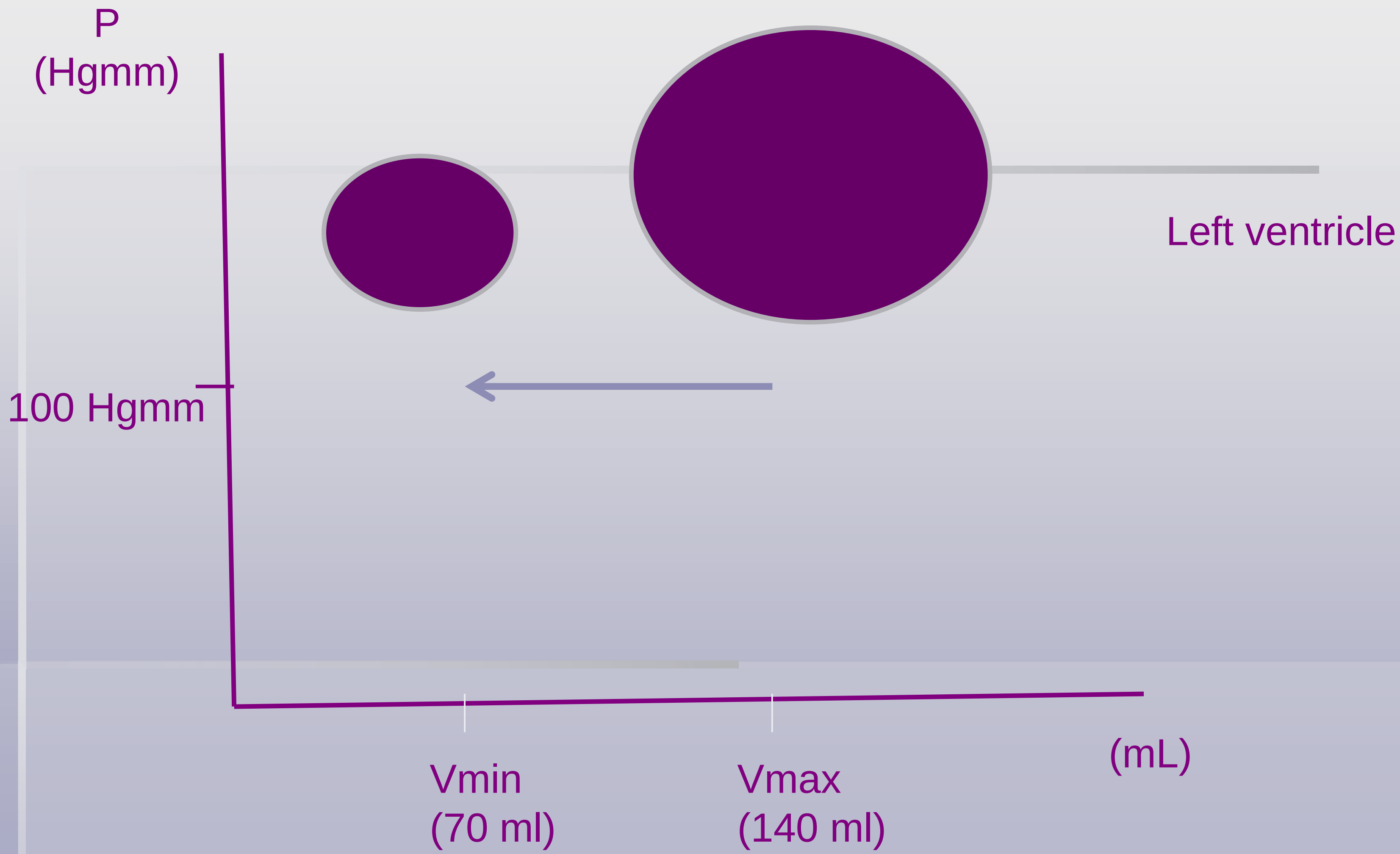
Left ventricle	Right ventricle
$p_m = 13.3$ kPa	$p_{m.} = 2.7$ kPa
$\Delta V = 70$ ml	$\Delta V = 70$ ml
$W = 0.93$ J	**$W = 0,19$ J**

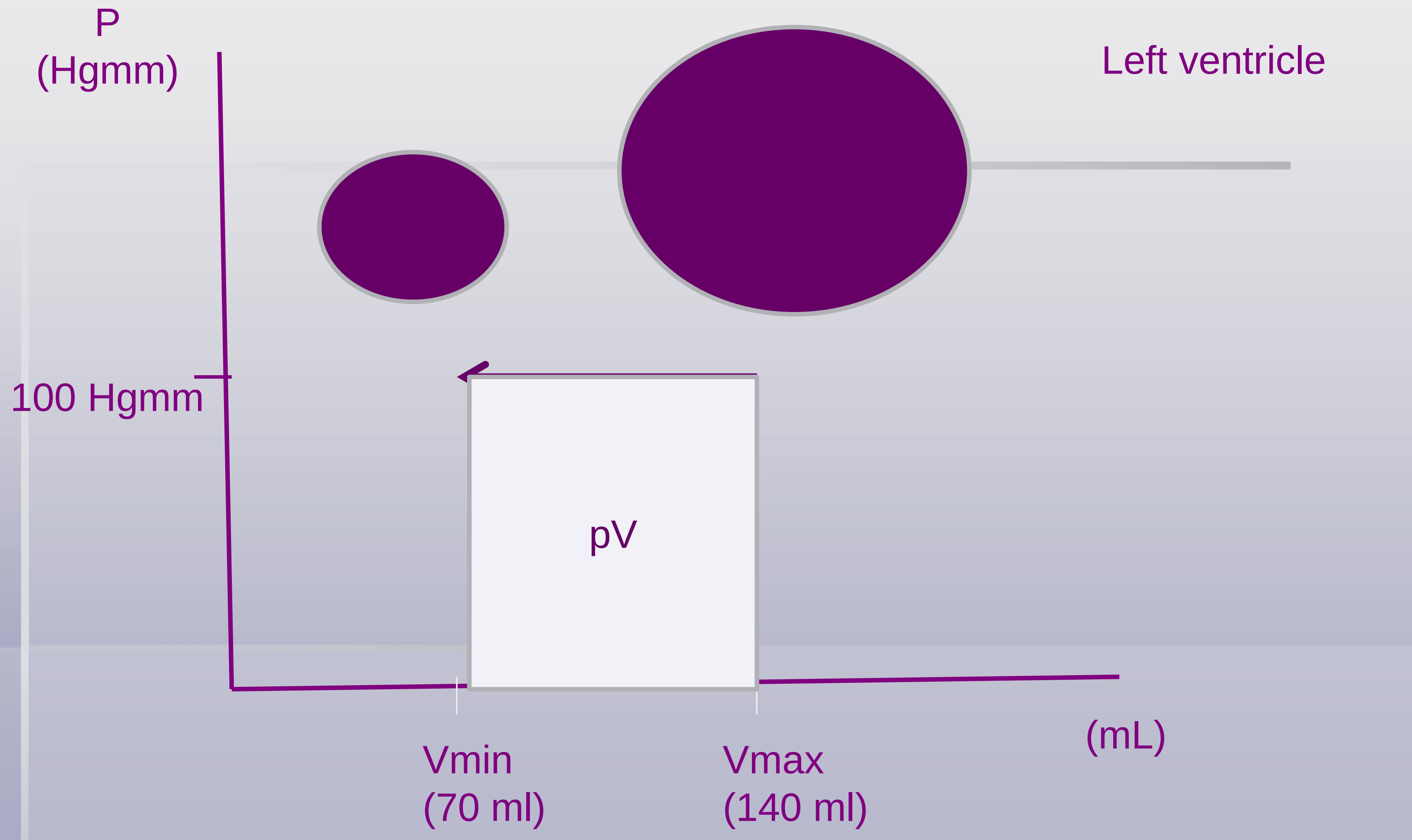
- *Mechanical energy of ejected blood volume W_k :*

Left ventricle	Right ventricle
= 0.009 J	**= 0,0018 J**

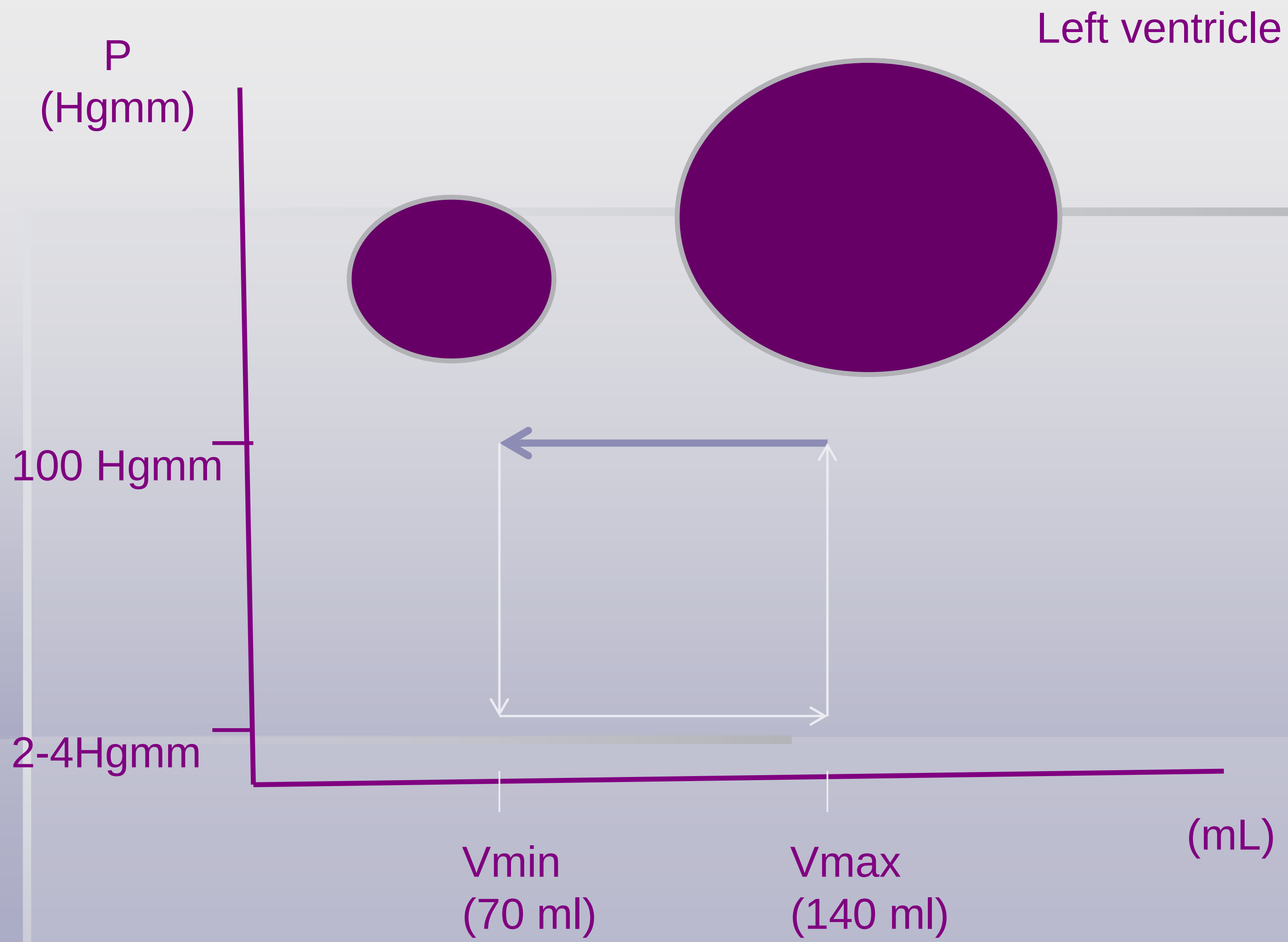
(since $W_k = 1/2.\rho.v_m^2.\Delta V$, $r = 1.06x10^3$ kg.m^{-3}, $v_{m.} = 0.3$ m.s^{-1}, resp. 0.22 m.s^{-1} in pulmonary artery)


Left ventricle
P
(Hgmm)
Vmin
Vmax
(140 ml)
(mL)


P
(Hgmm)
Left ventricle
100 Hgmm
Vmin
(70 ml)
Vmax
(140 ml)
(mL)



P
(Hgmm)
Left ventricle
100 Hgmm
pV
(mL)
Vmin
(70 ml)
Vmax
(140 ml)


Left ventricle
P
(Hgmm)
100 Hgmm
2-4Hgmm
Vmin
(70 ml)
Vmax
(140 ml)
(mL)